June 10, 2011

Here we explore the relation between the noncommutativity implied by $\mathcal{L} \cap \mathcal{G} \neq \emptyset$ and that implied by $[x^\mu, x^\nu] = i\Theta^{\mu\nu}$

$\mathcal{L}$ is a Lie Algebroid
$\mathcal{G}$ is a Lie Algebroid too.

generators of $\mathcal{L}$ are written d_e or p_a.
and satisfy

$$[d_e, d_m] = C_{em}^{\ \ n} d_n \qquad (1)$$

From
$$\{F, G\} = \frac{\partial F}{\partial x^\mu}\frac{\partial G}{\partial p_\mu} - \frac{\partial G}{\partial x^\mu}\frac{\partial F}{\partial p_\mu} + \frac{\partial F}{\partial p_a}\frac{\partial G}{\partial p_b} T_{ab}$$

What exactly is the nature of quantum corrections to the system of particles in a background gravitational field.

Tuesday 14th June 20~~10~~11

It is unclear to me what my goals should be for my summer work. In the last paper I wrote the equation of motion for spinless particles in a gravitational field in simple terms from commutators, following the inverse proof of Feynman's. Lately I found a simple Hamiltonian description for such particle which uses torsion in the symplectic form.

- Noether theorem for groupoids. * (tough)
- Birkhoff representations of the gauge gravity equations.
- The Lagrangian description and supersymmetry. (quite simple looking)
- Clean up the previous articles' typos.
- Can we reproduce the Bohr-Rosenfeld argument for the observability of electrodynamic fields to the case of gravitational fields?

How does noncommutativity enter gauge gravity?

$$[x^\mu, x^\nu] = i\theta^{\mu\nu} \longleftrightarrow \mathcal{L} \wedge \mathfrak{A} \neq 0$$

↑

Let's work on this today.

On noncommutativity:

Gelfand:

If $\mathfrak{A}$ is a commutative C^*-algebra, the set of non-zero homomorphisms $\mathfrak{A} \to \mathbb{C}$ called its spectrum $\hat{\mathfrak{A}}$ is a locally cmpct. topological space when considered as a subspace of the dual of $\mathfrak{A}$. Then $\hat{\mathfrak{A}}$ can be identified with X, and moreover $C(\hat{\mathfrak{A}}) = \mathfrak{A}$.

Heuristically:

$$t \in \hat{\mathfrak{A}} \qquad t : \mathfrak{A} \to \mathbb{C}$$

The identification of $t \in \hat{\mathfrak{A}}$ with an element $x \in X$ is given by equating the complex numbers:

$$t(f) = f(x). \quad \forall f \in \mathfrak{A}$$

$[x^\mu, x^\nu]_* = i\,\Theta^{\mu\nu}(x)$ (1)

$\{x^\mu\}$ complex numbers.

$(f * g)(x) = \cancel{f(x) *}$

$= \int f(k)\, g(k)\, E(k, x)\, dk$ (2)

$f \in \mathcal{O}$ $\hat{f}$ $\mathcal{A}$

$f \in \mathcal{O}$

$\hat{\mathcal{O}}$, the spectrum of $\mathcal{O}$ is

So, to build up $\mathcal{O}$, lets use $*$:

→ formal polynomials in $*$ products of coordinates x^μ.

$$f(x^\mu) = \sum_n c_n\, x_*^n$$

$\mathfrak{A} \cap \mathcal{D} \neq 0 \Rightarrow$ There are <u>inner</u> <u>derivations</u>.

$$\delta \in \mathcal{D} \rightarrow \qquad \delta(a_1 a_2) = (\delta a_1)\, a_2 + a_1 \,\delta(a_2) \qquad (3)$$

a derivation.

Act by commutation

$$[\delta, a_1 a_2] = a_1 [\delta, a_2] + [\delta, a_1]\, a_2 \qquad (4)$$

The elements of $\mathfrak{A}$ are denoted a_i and we assume they are nice smooth functions on some space X. Whether or not it is a genuine noncommutative space is determined by whether or not the product is a star product or an ordinary one.

pointwise

$$a_1 * a_2 \qquad \text{v.s.} \qquad a_1 \cdot a_2$$

From p 140 §6.4

As originally conceived, Gauge Theory starts with a wfn or field.

$$\psi \qquad (5)$$

and a connexion

$$A \qquad (6)$$

assigned to each point

$$p \in \cancel{\mathbb{X}}\ \mathbb{R}^4$$

$\mathbb{X}$ is spacetime.

So,

$$(\psi, A, \underset{\mathbb{R}^4}{\cancel{\mathbb{X}}})$$

1. But, $\mathbb{R}^4 \simeq \mathcal{T}(\mathbb{R}^4)$. I.e. $\mathbb{R}^4$ is isomorphic to its group of translations.
2. ψ and A admit alternative interpretations

   A. ψ can be considered as a rule specifying how its value changes under translations.

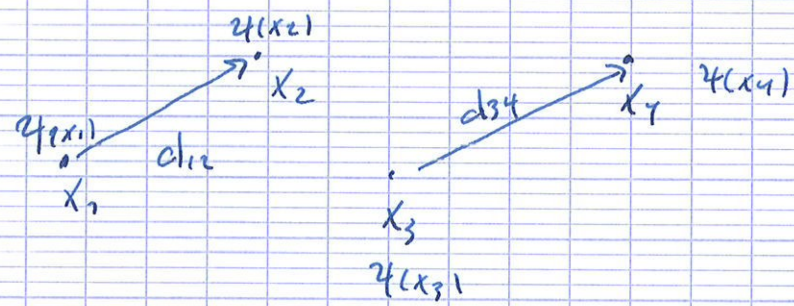
$\psi(x)$	x
$\psi(x_1)$	x_1
$\psi(x_2)$	x_2
$\psi(x_3)$	x_3
$\psi(x_4)$	x_4

x	$d \cdot x$
x_1	$d_{12} \cdot x_1 = x_2$
x_2	$d_{23} \cdot x_2 = x_3$
$\vdots$	
x_i	$d_{ij} \cdot x_i = x_j$

Instead of assigning a number $\in \mathbb{C}$ to each point we assign a number to each translation.

For each translation d_{ij} $x_i \to x_j$ we assign the number $\psi(x_j)$. or $\psi(x_i - x_j)$.

$$d_{ij}\, \psi(x_i) = \psi(x_j)$$



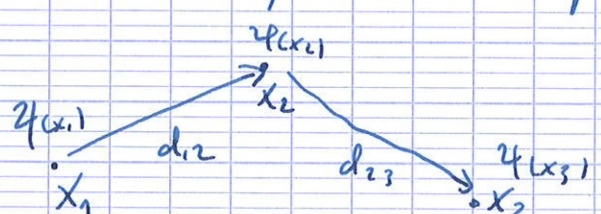
<u>How does ψ change under translations?</u>

$$d_{12}\psi(x_1) = \psi(x_2) - \psi(x_1)$$

$$d_{ij}\psi(x_i) = \underbrace{\psi(x_j) - \psi(x_i)}$$

If I know all these differences I can construct $\psi(x)$ and if I know $\psi(x)$ I can construct $\psi(x_j) - \psi(x_i)$ $\forall\, i,j$

E.g. Take a space with 3 points.



Say I know

1. $\psi_2 - \psi_1$
2. $\psi_3 - \psi_2$

I.e. 3 complex numbers. Then, call them.

(z_{ij}) $\quad i,j \in \{1,2,3\}$

$\downarrow$

3×3 complex symmetric matrix.

from this array of numbers we want to find z_1, z_2, z_3.

$$z_2 - z_1 = z_{12} \in \mathbb{C}$$
$$z_3 - z_2 = z_{23} \in \mathbb{C}$$
$$z_3 - z_1 = z_{13} \in \mathbb{C}$$

3 equations for 3 unknowns can be solved

$$\det \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{pmatrix} = -1 \begin{vmatrix} -1 & 1 \\ 0 & 1 \end{vmatrix} + 1 \begin{vmatrix} 0 & 1 \\ -1 & 1 \end{vmatrix}$$

$$= -1(-1) + 1(+1)$$

$$= 2 \neq 0$$

So knowing $d_{ij}\psi$ is the same as knowing $\psi(x_i)$ $\forall i$.

B. A is parallel transport of phase under translations

$$\psi(\xi') = \psi(\xi) + \varepsilon^l d_l \psi(\xi)$$

$$= (1 + \varepsilon^l d_l)\psi(\xi)$$

Once the operators d_l are given we no longer need to introduce the "points in space"

When there are no inner derivation:

* Regard elements of $\mathfrak{A} \otimes \mathcal{B}$ as matrices *

This is fundamental.

Let's call the indices upon which $\mathfrak{A}$ acts i, and those upon which ξ acts ξ.

$$a_{ij\xi\xi'}$$

$\{|i\rangle\}$ a basis for $\mathfrak{A}$

$\{|\xi\rangle\}$ a basis for $\mathfrak{B}$

$|i\rangle \otimes |\xi\rangle$

$\langle i|a(\xi,\xi')|j\rangle = a_{ij\xi\xi'}$

$\psi = \sum \underset{\downarrow}{\psi_i} |i\rangle + \psi_\xi |\xi\rangle \in \mathcal{H}$.

$= \cancel{\psi}$

$\psi = \psi_\sigma |\sigma\rangle \qquad \sigma = (i,\xi)$ a multi-index.

$a \in \mathfrak{A} \quad (a\psi)_{i\xi} = a_{ij}\psi_{j\xi} + a$

Say $a = \langle i| \underbrace{\langle\xi| \, a \, |\xi'\rangle}_{\downarrow\, a(\xi)} |j\rangle$

if a is diagonal

Wednesday June 18th 2011

Hilbert Space $\mathcal{H}$

We divide the basis into 2 parts

$$\mathcal{H} = \mathcal{H}_1 \oplus \mathcal{H}_2 \qquad (1)$$

$$\begin{array}{cc} \downarrow & \downarrow \\ |i\rangle & |\xi\rangle \end{array}$$

$$|\psi\rangle = \psi_i |i\rangle + \psi_\xi |\xi\rangle \equiv \psi_\sigma |\sigma\rangle \qquad (2)$$

Now, $a \in \mathcal{A}$, is diagonal in the $|\xi\rangle$ directions.

$$\Rightarrow \quad a_{ij\xi\xi'} = \langle\xi|\langle i|\, a\, |j\rangle|\xi'\rangle \qquad (3)$$

$$= \langle\xi|\, a_{ij}\, |\xi'\rangle = a_{ij}(\xi) \qquad (4)$$

Let's call the elements of $\mathcal{B}$, b. Let's assume that $\mathcal{B}$ acts upon $\mathcal{A}$ by commutation

$$b \circ a \equiv [b, a] = b.a - a.b$$

The product $a \cdot b$ is the usual matrix product.

The matrix elements of b:

$$\langle\xi|\langle i|\, b\, |j\rangle|\xi'\rangle = b_{ij\xi\xi'}$$

Now, we require b's to act on $\mathcal{O}$ via commutation. The b's should act on the ξ indices

$$b \leftrightarrow (1+\varepsilon^e d_e)$$

$(1+\varepsilon^e d_e)\xi = \xi'$ infl. displacement.

$$(1+\varepsilon^e d_e)\psi(\xi) = \psi(\xi')$$

$$d_e = e_e{}^\mu \partial_\mu$$

$\psi(\xi)$ is a wfn. $\leftrightarrow$ $\boxed{\psi(\xi)}|\xi\rangle$

$$(1+\varepsilon^e d_e)\,|\psi\rangle$$

$= (1+\varepsilon^e d_e)\left(\psi_i\,|i\rangle + \psi_\xi\,|\xi\rangle\right)$ summation

$$= |\psi\rangle + \varepsilon^e d_e\left(\psi_i\,|i\rangle + \psi_\xi\,|\xi\rangle\right)$$

$$\cancel{\varepsilon^e d_e \psi_\xi \to \varepsilon^e}$$

$$(1+\varepsilon^e d_e)|\psi\rangle = \psi_i\,|i\rangle + \psi_{\xi'}\,|\xi\rangle$$

$$\int \psi(\xi')\,|\xi\rangle\,\boxed{d\xi}$$

What is the local translation invariant measure ????

$$(1 + \varepsilon^e e_e{}^\mu \partial_\mu)\psi(\xi)\,|\xi\rangle$$

Let's forget about the i indices for a moment and just review displacements in $|\xi\rangle$ space.

$$\mathcal{H} = \{\psi \mid \psi = \psi_\xi |\xi\rangle \text{ sum}\}$$

$$\int \psi_\xi\, |\xi\rangle\, d\mu(\xi) = |\psi\rangle$$

$$b = (1+\varepsilon^e d_e) = (1+\varepsilon^e e_e{}^\mu \partial_\mu)$$ - displacement element.

satisfying ~~Eb~~ $[d_e, d_m] = C_{em}{}^n d_n$

so now I need to find some 'teleparallel equivalents' of common 3D Euclidean solutions.

Aside:

Considering

Thursday 16th 2011

Let's look @ some 'teleparallel' equivalents of some Simple 2D, 3D, 4D Euclidean spacetimes.

Aside: The usual reason for dealing with a preferred connexion is analytic. In other words, We say the 2-sphere has nonzero curvature because it admits no connexion <u>that</u> <u>is</u> <u>non-singular</u> and produces a zero curvature. These types of results are in differential topology. If ~~however~~ we are dealing from the start with a ~~spacetime~~ geometric gravity theory, we expect to deal very generally with singularities. This leads one to wonder if the natural constraints of differential topology really should restrict the choice of connexion. In this vein it is quite natural to consider singular connexions.

$$K^{\lambda}{}_{\mu\nu} = \frac{1}{2}\left[T^{\lambda}{}_{\mu\nu} + T_{\mu\nu}{}^{\lambda} + T_{\nu\mu}{}^{\lambda}\right]$$

$$K^{\lambda}{}_{\mu\nu} = \overset{\circ}{\Gamma}{}^{\lambda}{}_{\mu\nu} - \Gamma^{\lambda}{}_{\mu\nu}$$

For the sphere

1. $\overset{\circ}{\Gamma}$ can be calculated
2. $\Gamma \sim h \partial h$ can be calculated
3. Thus K can be computed
4. Torsion can be computed.

Friday 17th 2011

TO DO:

1. Make tables of contents
2. Scan notebooks
3. Organize my project on generalized gauge theory.
4. Organize computer

June 24^{th} 2011

It should be possible to classify the <u>dimension</u> of extended objects introduced into generalized gauge gravity via

$$\mathfrak{A} \cap \mathfrak{L} \neq \emptyset \qquad (1)$$

by asking how many linearly independent displacements are simultaneously gauge transformations.

On the introduction of the gauge potentials in both the Yang-Mills and gravitational case within the Feynman scheme:

$$YM \rightarrow \quad (i\hbar)^{-1}[p^\mu, I^a] =: \breve{A}^{\mu a}(x, q, I) \qquad (2)$$

$$\breve{A}^{\mu a} = \underbrace{\mathcal{A}^\mu{}_c (\breve{I}^c)^a{}_b}_{(\breve{\mathcal{A}}^\mu)^a{}_b \;\leftarrow\; \underline{\text{adjoint representation}}} I^b \qquad (3)$$

In words, the Yang-Mills gauge potl. is defined by the behavior of the color vector under a spacetime translation.

When one translates a color vector I^a to a nearby point $\delta_T I^a$, the new vector is rotated in the new vectors space.

the amount of this rotation is determined exactly by the gauge field. That is the meaning of (2), (3).

If we examine (2), (3) and replace

$$I^a \to p^a \qquad \text{III}$$

$$[\dot{x}^\mu, p^a]$$

$$[p^\mu, p^a] = [p^\mu, p^\nu h^a{}_\nu]$$

$$= [p^\mu, p^\nu] h^a{}_\nu + p^\nu [p^\mu, h^a{}_\nu]$$

$$= \underset{(-i\hbar)}{\underset{\sim}{\;}} W_q^{\mu\nu} h^a{}_\nu + p^\nu \underbrace{(i\hbar)\,\partial^\mu h^a{}_\nu}$$

p here we can write ∂h in terms of connexions and combine with the first term.

$$\partial^\mu h^a{}_\nu$$

$$\cancel{h_{a\lambda}\,\partial^\varepsilon h^a}$$

p

p

p

To whip this into interpretable shape we use

$$h^a{}_\lambda \, \partial^\mu h_{a\varepsilon} = g_{\lambda\eta} \, g^{\mu\delta} \, \Gamma^\eta{}_{\varepsilon\delta}$$

$$h_{a\lambda} \, \partial^\mu h^a{}_\varepsilon = \dots$$

$$\underbrace{h^{b\lambda} h_{a\lambda}}_{\substack{\cancel{\eta_{ba}} \\ \delta^b{}_a}} \partial^\mu h^a{}_\varepsilon = h^{b\lambda} g_{\lambda\eta} \, g^{\mu\delta} \, \Gamma^\eta{}_{\varepsilon\delta}$$

$$\partial^\mu h^b{}_\varepsilon = h^b{}_\eta \, g^{\mu\delta} \, \Gamma^\eta{}_{\varepsilon\delta}$$

In our case

$b \to a$

$\cancel{\nu \to \varepsilon}$

$\varepsilon \to \nu$

$$\partial^\mu h^a{}_\nu = h^a{}_\eta \, g^{\mu\delta} \, \Gamma^\eta{}_{\nu\delta}$$

So, from the previous page

$$[\tilde{p}^\mu, \tilde{p}^a] = -i\hbar \, \backslash X / g^{\mu\nu} h^a{}_\nu + \underbrace{p^\nu (i\hbar) \, h^a{}_\eta \, g^{\mu\delta} \, \Gamma^\eta{}_{\nu\delta}}_{\checkmark}$$

Now.

$$|X|_{cy}^{\mu\nu} = F_{cy}^{\mu\nu} - g^{\sigma\delta} g^{\varepsilon\eta} \Gamma^{\lambda}_{\eta\delta} p_\varepsilon$$

$$- g^{\sigma\delta} g^{\lambda\eta} \Gamma^{\varepsilon}_{\eta\delta} p_\varepsilon$$

$$+ g^{\lambda\delta} g^{\varepsilon\eta} \Gamma^{\sigma}_{\eta\delta} p_\varepsilon$$

$$+ g^{\lambda\delta} g^{\sigma\eta} \Gamma^{\varepsilon}_{\eta\delta} p_\varepsilon$$

Working like

$$[p^\mu, \pi^a] = (-i\hbar)\Big\{ F_{cy}^{\mu\nu} h^a_{\ \nu} - g^{\sigma\delta} g^{\varepsilon\eta} \Gamma^{\lambda}_{\eta\delta} p_\varepsilon h^a_{\ \nu}$$

$$- g^{\sigma\delta} g^{\lambda\eta} \Gamma^{\varepsilon}_{\eta\delta} p_\varepsilon h^a_{\ \nu}$$

$$+ g^{\lambda\delta} g^{\varepsilon\eta} \Gamma^{\sigma}_{\eta\delta} p_\varepsilon h^a_{\ \nu}$$

$$+ g^{\lambda\delta} g^{\sigma\eta} \Gamma^{\varepsilon}_{\eta\delta} p_\varepsilon h^a_{\ \nu} \Big\}$$

$$+ (i\hbar)\, \underbrace{p^\nu}_{} h^a_{\ \eta}\, g^{\mu\delta} \Gamma^{\eta}_{\nu\delta}$$

$$g^{\nu\varepsilon} p_\varepsilon$$

last term: $\boxed{g^{\nu\varepsilon} g^{\mu\delta} \Gamma^{\eta}_{\nu\delta} p_\varepsilon h^a_{\ \eta}}$ **

let $\nu \to \eta$

$\eta \to \sigma$

$$g^{\eta\varepsilon} g^{\mu\delta} \Gamma^{\sigma}_{\eta\delta} p_\varepsilon h^a_{\ \nu}$$

So in the absence of electrodynamic forces

$$[p^\mu, p^a] = (i\hbar) \{ g^{a\delta}$$

The basic form is

$$g^2 \Gamma h p$$

$$+ g^2 \Gamma h p$$

$$- g^2 \Gamma h p$$

$$- g^2 \Gamma h p$$

$$+ g^2 \Gamma h p$$