Sat. June 9th 2012

Some notes on Lie's third theorem.

This theorem claims that one can essentially reconstruct a Lie Group directly from the structure constant. Here I will follow the argument in H. Flanders that sees Lie's third theorem as a result in the theory of partial differential equations.

First, we work in E^n & introduce n^3 constants satisfying:

$$C^i_{(jk)} = 0 \quad \text{- antisymmetry} \tag{1}$$

$$C^i_{jk} C^j_{rs} + C^i_{js} C^j_{kr} + C^i_{jr} C^j_{sk} = 0 \quad \text{Jacobi} \tag{2}$$

Lie's Third Theorem ~~implies~~ asserts the existence of n linearly independent one forms ~~$E d\sigma^i$~~ $\{\sigma^i\}_{i=1}^n$ defined on some nbd. of the origin $0 \in E^n$, and furthermore satisfying

$$d\sigma^i = \frac{1}{2} \sum C^i_{jk} \sigma^j \wedge \sigma^k \tag{3}$$

on the nbd, N_0, of the origin.

June 13th 2012

T inv. under $\bar{V}, \bar{W}$

$$\mathcal{L}_{\bar{V}} T = 0 \qquad (1)$$

$$\mathcal{L}_{\bar{W}} T = 0 \qquad (2)$$

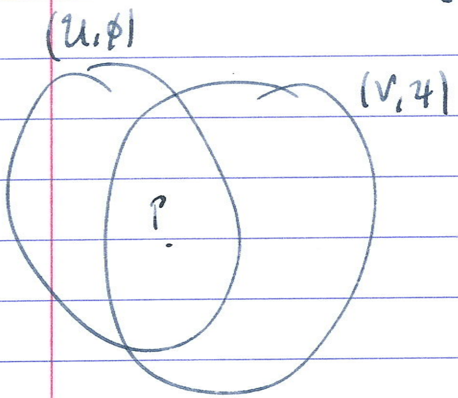
If a and b are constants on the manifold, T is invariant under the field $\bar{Z} := a\bar{V} + b\bar{W}$.

$$\mathcal{L}_{\bar{Z}} T = 0 \qquad (3)$$

$$\mathcal{L}_{\bar{V}+\bar{W}}$$

If we treat $\mathbb{C}$ as a $\left[{1 \atop 2}\right]$ tensor define in the nbd. of some point $p \in M$, what would happen to such a $\mathbb{C}$ if a coordinate change is made?

$$\mathbb{C}(P) = (C^i{}_{jk}) \longmapsto \underbrace{J^i{}_l \, J_j{}^m \, J_k{}^n} \, C^l{}_{mn}$$



these are Jacobians induced by the change of coordinates. Since they are evaluated at P they are: $(J^i{}_l(P)) \in Gl(n,\mathbb{R})$ and therefore correspond to a change of basis in the Lie Algebra, leaving the Lie Algebra intact.

June 15^{th} 2012

Generalized gauge structures

I. Classical Electrodynamics ($U(1)$ gauge theory)

i) $(\mathbb{R}^4, \eta)$ - spacetime (background)

ii) $U(1)$ - group (gauge)

$\rightarrow$ Fibre bundle with base space $(\mathbb{R}^4, \eta)$ and structure group $U(1)$

II. Yang-Mills theory (G compact semisimple gauge theory)

generalizes above (EM) in its second ~~argument~~ ingredient

i) $(\mathbb{R}^4, \eta)$ - background spacetime

ii) G - structure group (compact semisimple)

III. Group manifold as base space

July 23rd 2012

Notes on 2 papers on groupoids

i) A. Weinstein Groupoids: Unifying internal and external symmetry [1]
ii) R. Brown From groups to groupoids: a Brief survey. [2]

We first examine the definition and motivating example in [1], the semi-tiled plane.

A <u>groupoid</u> with base B ~~and~~ is a set G with mappings $\alpha, \beta : G \to B$ and a partially defined binary operation $(g,h) \to gh$ satisfying the following 4 axioms:

1. The operation is defined only for certain pairs of elements:
   gh is only defined when $\beta(g) = \alpha(h)$
2. It is <u>associative</u>: if either $(gh)k$ or $g(hk)$ is defined the other is as well and they are equal.
3. For each $g \in G$ there are left and right identities λ_g, ρ_g s.t. $\lambda_g g = g$ and $g \rho_g = g$.
4. Each g in G has an inverse g^{-1} for which $\cancel{gg^{-1} = \lambda_g}$
   $gg^{-1} = \lambda_g \qquad g^{-1}g = \rho_g$

From a categorical perspective, elements of G are arrows and elements of B are objects: arrows are multiplied by putting them tip to tail.

An extremely trivial theorem is: $\alpha(g^{-1}) = \beta(g)$. This follows simply because gg^{-1} is defined.

We can translate this to a category diagram

$$\begin{tikzcd}
\alpha(g)=\alpha(gh) & \beta(g)=\alpha(h) \arrow[l, bend right, "g"'] & \beta(g)=\beta(gh) \arrow[l, bend right, "h"'] \arrow[ll, bend right, "gh"']
\end{tikzcd}$$

tip-to-tail: first h then g.

Note: $\alpha(gh) = \alpha(g)$

pf: $\cancel{\alpha(ghh^{-1}) = \alpha(g f_h) = \alpha(g)}$

$(\lambda_g g)h = \lambda_g(gh)$ implies that $\lambda_g(gh)$ is defined since the LHS is obviously defined.

The idea is simple <u>gh is defined only when the target of h is the source of g.</u> Thus α is the target map and β is the source map.

This is kind of similar to requiring the range element of a certain map to live in the domain of another in order for the composition of those maps to be defined on that element.

Now, going back to $\alpha(gh) = \alpha(g)$: its obvious because the target of gh is the same as the target of g.

Now we try to show that $\alpha(g)$ and λ_g determine eachother.

~~$\alpha(g)$ is the target map of g~~ e

$\alpha(g)$ is the target map ($\alpha: G \to B$) evaluated at g, meaning the Point in B upon which ~~map~~ the map (arrow) g lands. We WTS that this point determines the arrow that corresponds to the identity of g. To see this, perhaps.

$$\alpha(\lambda_g g) = \alpha(g) \quad \text{by the theorem on the prev. page}$$

$$\cancel{\alpha(\lambda_g) \text{ by the thm}}$$

$$\alpha(g) \quad \text{by the definition of } \lambda_g \text{ (left identity).}$$

since $\alpha(\lambda_g g h) = \alpha(g) \ \forall h.$

Suppose I know $\alpha(g)$, how can I find λ_g? Well, λ_g must satisfy $\alpha(\lambda_g g) = \alpha(g)$

$$\alpha(gg^{-1}) = \alpha(g^{-1}) = \alpha(\lambda_g)$$

$$\alpha(gg^{-1}) = \cancel{\alpha(g)}$$

$$\cancel{\alpha(g^{-1}g) = \alpha(g) = \alpha(\lambda_g)}$$

OK... so $\alpha(g^{-1}g) = \alpha(g)$ ~~and~~ So $\alpha(g) = \alpha(\lambda_g)$

$$\alpha(g^{-1}g) = \alpha(\lambda_g)$$

So I know $\alpha(g)$ at least defines $\alpha(\lambda_g)$; does $\alpha(\lambda_g)$ define λ_g?

So, $\alpha(gg^{-1}) = \alpha(\lambda_g)$.

$$\begin{array}{c} \alpha(gg^{-1}) = \alpha(\lambda_g) \\ \| \quad /\!/ \\ \alpha(g) \end{array} \implies \alpha(g) = \alpha(\lambda_g) \;\; \forall g \in G.$$

The target of an arrow must be the target of its left identity

So suppose I wanted to figure out λ_g, but only knew $\alpha(g)$. Well, g^{-1} is the reversed arrow of g and if I knew that I can compute $gg^{-1} = \lambda_g$, so I can just find g^{-1} from $\alpha(g)$ but

Maybe a diagram helps:

$$\begin{tikzcd}
\alpha(gg^{-1}) & \alpha(g) = \beta(g^{-1}) \arrow[l, bend right, "g"'] & \beta(g^{-1}) = \beta(gg^{-1}) \arrow[l, bend right, "g^{-1}"'] \arrow[ll, bend right=50, "gg^{-1}"']
\end{tikzcd}$$

Then

$$\begin{tikzcd}
\alpha(\lambda_g) & \alpha(\lambda_g) = \beta(\lambda_g) \arrow[l, bend right, "g"'] & \beta(\lambda_g) \arrow[l, bend right, "g^{-1}"'] \arrow[ll, bend right=50, "\lambda_g"']
\end{tikzcd}$$

Note: Since

1. $\beta(gg^{-1}) = \beta(g^{-1})$
   $\|$
   $\beta(\lambda_g)$

2. Since $\alpha(g) = \beta(g^{-1})$
   $\|$
   $\alpha(\lambda_g) = \beta(\lambda_g)$

Now since $\alpha(\lambda_g) = \beta(\lambda_g)$ all three bottom base points are the same

Notes on the teleparallel equivalent of General Relativity.

From a paper by Pereira et. al.

"Whether gravitation a curved or torsioned spacetime — or equivalently a Riemannian or a Weitzenböck spacetime structure — turns out to be, at least classically, a matter of convention."

This was exactly Poincaré's point of view, but given the amazing success of GR circa 1920's and the fact that Poincaré did not have the precise example of teleparallel theory in hand, it is unfortunate but understandable that his very deep insight lost traction. Even in the 1980's, as evidenced by the arguments in the wonderful book of Torretti, the view that the choice of geometry cannot be conventional as Poincaré had insisted was still dominant. At this point (2012) the force of Poincaré's insight seems to be apparent. GR could have been discovered through teleparallel gravity — it is not at all and awkward reformulation that could only have been devised once one already knew the answers from ordinary GR, which is basically the charge that advocates of Poincaré's conventionalism have had to face.

$$B_\mu^{\bullet} = B^a{}_\mu \mathcal{P}_a \qquad (1)$$

B_μ is, in Pereira's conception, the gauge potential of the gravitational field. Its values lie in the Lie Algebra of the translation group.

In my conception the values would lie in the Lie Algebroid of the translation groupoid.

For Pereira

$$[\mathcal{P}^a, \mathcal{P}^b] = 0 \qquad (2) *$$

I believe this is simply wrong. The translation algebroid is <u>non-abelian</u>.

$$\mathcal{P}^a = \partial/\partial x^a \qquad (3) *$$

The status of this equation is a bit unclear in my formalism. The status of x^a in my theory is very similar (perhaps formally equivalent) to conceptions in which x^a are Goldstone fields (c.f. Fasguerra) But the space upon which these scalars are a parametrization must be considered with care: Is it an affine space e.g.?

In my theory (2)* should be replaced by

$$[\mathcal{P}^a_x, \mathcal{P}^b_x] = T_x{}^{ab}{}_c \, \mathcal{P}^c_x \qquad (2)$$

where

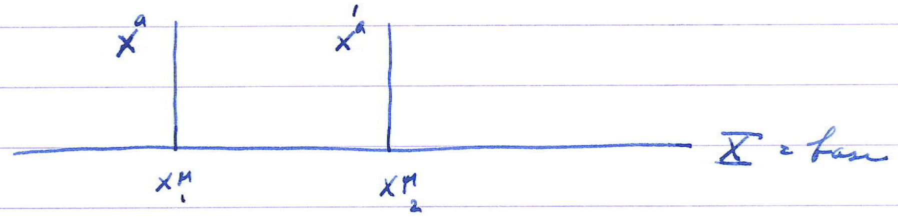
$T^{ab}{}_c(x)$ is basically the gravitational field.

For Pereira, a gauge transformation is

$$x^{a'} = x^a + \alpha^a \qquad \alpha^a(x^\mu) \qquad (4)*$$

$$\delta x^a = (\alpha^b P_b) \cdot x^a \qquad (5)*$$

Here Pereira defines a gauge transformation to be a translation of the parameters that parametrize the tangent space, but requiring a different parameter translation at different points in the spacetime manifold. E.g.



2 pts.

We allow a translation, e.g. $x^a + 5$ @ x^μ_1 to be different from the one @ x^μ_2, e.g. $x^a + 7$.

Now consider a general source field $\Psi(x^\mu)$, defined on the base manifold. It may be a vector scalar or spinor. But regardless of its internal structure it has the gauge transformation

$$\delta\Psi = (\alpha^a P_a) \cdot \Psi \qquad (6)*$$

$\delta\Psi$ stands for the functional change at the pt x^μ (for a clear definition of functional change we refer the reader to Blagojevich).

Pereira makes a point of stating that the translation generators are able to act on the argument of any source field because

$$\partial_\mu = (\partial_\mu x^a)\,\partial_a \quad \text{and} \quad \partial_a = (\partial_a x^\mu)\,\partial_\mu \qquad (7)*$$

but these relations ~~look~~ look extremely suspicious to me. Per. seems to be glossing over some essentially critical points in the definition of the alternative tangent space parametrized by the x^a's.

This leads him to

$$\cancel{h^\mu = \partial}$$

$$h_\mu = \partial_\mu + B^a{}_\mu \frac{\delta}{\delta x^a} \qquad (8)*$$

strange use of the functional derivative symbol $\frac{\delta}{\delta x^a}$. I think at this point he is pretty confused and just wishes to obfuscate things.

$$h_\mu \psi = \partial_\mu \psi + (B^a{}_\mu P_a)\cdot\psi \qquad (9)*$$

$$= (h^a{}_\mu P_a)\cdot\psi \qquad (10)*$$

$$h^a{}_\mu = \partial_\mu x^a + B^a{}_\mu \equiv h_\mu x^a \qquad (11)*$$

anholonomic.

a dubious definition

~~General Idea~~

Take a look at some teleparallel solutions:

- de Sitter
- Schwarzschild Solution
- Kerr-Newman

de Sitter

the teleparallel version of dS is a <u>constant torsion maximally symmetric</u>

It is a solution of the sourceless teleparallel field equation with a cosmological term Λ:

○ ≡ GR
● ≡ teleparallel

$$\partial_\sigma (h \overset{\bullet}{S}_\lambda{}^{\rho\sigma}) - k^2 (h \overset{\bullet}{t}_\lambda{}^\rho) - \delta_\lambda{}^\rho \Lambda = 0$$

Go to · stereographic coordinates
· one specific class of frames

$$h^a{}_\mu = \Omega \, \delta^a{}_\mu \; ; \quad \Omega := \frac{1}{1 - (\sigma^2 \Lambda / 12)}$$

$$\sigma^2 = \eta_{\mu\nu} x^\mu x^\nu \qquad g_{\mu\nu} = h^a{}_\mu h^b{}_\nu \eta_{ab} = \Omega^2(x) \eta_{\mu\nu}$$

Weitzenböck connection:

$$\overset{\bullet}{\Gamma}{}^\rho{}_{\nu\mu} := h_a{}^\rho \partial_\mu h^a{}_\nu$$

$$\cancel{h^a{}_\mu = \Omega\,\delta^a{}_\mu \quad \text{thus,}}$$

$$\cancel{\overset{\bullet}{\Gamma}{}^\rho{}_{\nu\mu} = \Omega\,\delta_a{}^\rho\,\partial_\mu \Omega\,\delta^a{}_\nu}$$

$$\cancel{= \Omega\,\delta^\rho{}_\nu\,\partial_\mu \Omega}$$

$$\sigma^2 = \eta_{\mu\nu} x^\mu x^\nu$$

$$\Omega = \frac{1}{1-(\sigma^2\Lambda/12)}$$

$$\partial_\mu \ln \Omega$$

$$= -\partial_\mu \left(1 - \frac{\Lambda}{12}\sigma^2\right)$$

$$= \frac{\Lambda}{12}\partial_\mu \sigma^2$$

$$= \frac{\Lambda}{12}\left(\partial_\mu \eta_{\sigma\rho} x^\sigma x^\rho\right)$$

$$= \frac{\Lambda}{12}\eta_{\sigma\rho}\left(\delta_\mu{}^\sigma x^\rho + x^\sigma \delta_\mu{}^\rho\right)$$

$$= \frac{\Lambda}{12}\left(x_\mu + x_\mu\right) = \frac{\Lambda}{6} x_\mu$$

$$h^a{}_\mu = \Omega\,\delta^a{}_\mu$$

$$\overset{\bullet}{\Gamma}{}^\rho{}_{\nu\mu} = \underbrace{h_a{}^\rho}_{\Omega^{-1}\delta_a{}^\rho}\partial_\mu \underbrace{h^a{}_\nu}_{\Omega\,\delta^a{}_\nu}$$

ln

$$= \Omega^{-1}\underbrace{\delta_a{}^\rho\,\delta^a{}_\nu}_{\delta^\rho{}_\nu}\partial_\mu \Omega$$

$$\cancel{= \delta^\rho{}_\nu\,\partial_\mu \ln}$$

$$= \delta^\rho{}_\nu\,\partial_\mu \ln \Omega \qquad \text{agrees with Eq 7.5.}$$

Torsion: $$\overset{\bullet}{T}{}^\lambda{}_{\mu\nu} = \delta^\lambda{}_\nu\,\partial_\mu \ln\Omega - \delta^\lambda{}_\mu\,\partial_\nu \ln\Omega$$

$$= \delta^\lambda{}_\nu \frac{\Lambda}{6} x_\mu - \delta^\lambda{}_\mu \frac{\Lambda}{6} x_\nu$$

$$= 2\frac{\Lambda}{6}\,\delta^\lambda{}_{[\nu} x_{\mu]} \qquad \text{(the two is from def. of } [\nu,\mu])$$

$$\boxed{\overset{\bullet}{T}{}^\lambda{}_{\mu\nu} = \frac{\Lambda}{3}\,\delta^\lambda{}_{[\nu} x_{\mu]}}$$

So we have $\dot{T}$ a linear function in x. This is suppose to be "constant torsion". So we need to understand constant torsion

$$\dot{S}^{\rho\mu\nu} = \left[\dot{K}^{\mu\nu\rho} - g^{\rho\nu} \dot{T}^{\sigma\mu}{}_{\sigma} + g^{\rho\mu} \dot{T}^{\sigma\nu}{}_{\sigma} \right]$$

The Lagrangian can be written in terms of this "Superpotential"

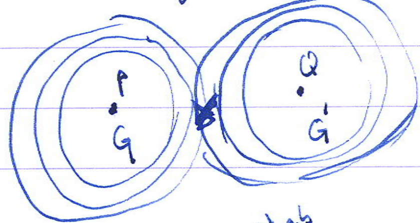
$$\dot{\mathcal{L}} = \frac{h}{4k^2} \dot{T}_{\rho\mu\nu} \dot{S}^{\rho\mu\nu}$$

Thinking again about the big picture

GR with an asymptotic group allows the definition of a Hamiltonian especially when there is SUSY.

But if we break up GR into peices that have certain asymptotic symmetries we lose some very important insights about the theory of gravitation. The typical solution of GR (M, g) does not possess any symmetry! The absence of a global timelike killing vector $\Rightarrow$ no Hamiltonian can be written. To my mind this is the fundamental reason for the inability to quantize. With no group around what do we do?

Answer - Groupoids. - Algebroids



$[\Upsilon^a, \Upsilon^b] = [T \cdot \Upsilon]^{ab}$

$[\Upsilon^a, \Upsilon^b] = T^{ab}{}_c \Upsilon^c$ - is an algebra - what symmetry does it define?

- local translation
  - → actually it must be more specific. What kind of local translation? Is there a local integration to a group?

$$[T \cdot \Upsilon]^{ab} \cdot \Psi = \delta \Psi \Big|_{P \in M}$$

ε_{ijk} $\quad SU(2)$

$SU(3)$

$$\delta^c{}_b \bar{X}^a{}_d - \delta^a{}_d \bar{X}^c{}_b$$

$$\cancel{\delta^c{}_b \delta^f{}_e}$$

$$\underbrace{\left(\delta^c{}_b \delta^a{}_f \delta^g{}_d - \delta^a{}_d \delta^c{}_f \delta^g{}_b\right)} \bar{X}^f{}_g$$

$$\left(\delta^c{}_b \delta^a{}_f \delta^g{}_d - \delta^c{}_f \delta^a{}_d \delta^g{}_b\right) \bar{X}^f{}_g$$

Certain fns $T^{\gamma}{}_{\mu\nu}(x)$ correspond to certain local symmetries in a nbd. of the pt $x \in \Sigma$.

For dS space $\dot{T}^{\lambda}{}_{\mu\nu} = \frac{\Lambda}{3}\delta^{\lambda}{}_{[\nu}x_{\mu]}$ in the torsion.

Divergence of torsion

$$\cancel{\partial^{\mu}\dot{T}^{\lambda}{}_{\mu\nu} = \partial^{\mu}\left(\frac{\Lambda}{6}\right)(\delta^{\lambda}{}_{\nu}x}$$

$$\partial^{\mu}\dot{T}^{\lambda}{}_{\mu\nu} = \frac{\Lambda}{6}\partial^{\mu}(\delta^{\lambda}{}_{\nu}x_{\mu} - \delta^{\lambda}{}_{\mu}x_{\nu})$$

$$= \frac{\Lambda}{6}(\delta^{\lambda}{}_{\nu}\cdot 4 - \delta^{\lambda}{}_{\mu}\delta^{\mu}{}_{\nu})$$

$$= \frac{\Lambda}{6}(4\delta^{\lambda}{}_{\nu} - \delta^{\lambda}{}_{\nu}) = \frac{\Lambda}{2}\delta^{\lambda}{}_{\nu}$$

For curvaturure Rieimann → Ricci → scalar.
" torsion

$$\dot{T}^{\lambda}{}_{\lambda\nu} = \frac{\Lambda}{6}(\delta^{\lambda}{}_{\nu}x_{\lambda} - \delta^{\lambda}{}_{\lambda}x_{\nu})$$

$$= \frac{\Lambda}{6}(x_{\nu} - 4x_{\nu}) = -\frac{\Lambda}{2}x_{\nu}$$

Teleparallel dS:

$$\dot{T}^{\lambda}{}_{\mu\nu} = \frac{\Lambda}{3}\, \delta^{\lambda}{}_{[\nu} X_{\mu]} \qquad \text{torsion for dS} \qquad (1)$$

$$\partial^{\mu} \dot{T}^{\lambda}{}_{\mu\nu} = \frac{\Lambda}{2} \delta^{\lambda}{}_{\nu} \qquad \text{div } T \text{ for dS} \qquad (2)$$

$$\dot{T}^{\lambda}{}_{\lambda\nu} = -\frac{\Lambda}{2} \cancel{\#} X_{\nu} \qquad \dot{T}_{\nu} \text{ for dS} \qquad (3)$$

The Lagrangian density for ~~dS~~ Teleparallel is

$$\dot{\mathcal{L}} = \frac{h}{2k^2}\left[\frac{1}{4}\dot{T}^{\rho}{}_{\mu\nu}\dot{T}_{\rho}{}^{\mu\nu} + \frac{1}{2}\dot{T}^{\rho}{}_{\mu\nu}\dot{T}^{\nu\mu}{}_{\rho} - \dot{T}_{\rho\mu}{}^{\rho}\dot{T}^{\nu\mu}{}_{\nu}\right] \quad (4)$$

$$k^2 = \frac{4\pi G}{c^4} \qquad h = \det(h^a{}_{\mu})$$

$$\dot{\mathcal{L}}_{dS} = \frac{h}{2k^2}\left[\frac{1}{4}\cdot\left(\frac{\Lambda}{3}\right)^2 \delta^{\rho}{}_{[\nu} X_{\mu]}\, \delta_{\rho}{}^{[\nu} X^{\mu]} + \cdots - \cdots\right] \quad (5)$$

Explicitly this first term is:

$$\frac{1}{4}\cdot\left(\frac{\Lambda}{3}\right)^2\cdot\frac{1}{4}\cdot\left(\delta^{\rho}{}_{\nu} x_{\mu} - \delta^{\rho}{}_{\mu} x_{\nu}\right)\left(\delta_{\rho}{}^{\nu} x^{\mu} - \delta_{\rho}{}^{\mu} x^{\nu}\right) \quad (6)$$

$$\left(\frac{1}{4}\right)^2\left(\frac{\Lambda}{3}\right)^2\cdot\left\{4x^2 - \underbrace{\delta_{\nu}{}^{\mu} x_{\mu} x^{\nu}}_{x^2} - \underbrace{\delta^{\nu}{}_{\mu} x_{\nu} x^{\mu}}_{x^2} + 4x^2\right\} \quad (7)$$

$$= \left(\frac{1}{4}\right)^2\left(\frac{\Lambda}{3}\right)^2 \cdot 14 \cdot x^2 \quad (8)$$

Now, $\dot{T}^{\nu\mu}{}_{\rho}$ = raise middle index.

$$g^{\varepsilon\mu}\dot{T}^{\nu}{}_{\varepsilon\rho} = g^{\varepsilon\mu}\cdot\frac{\Lambda}{3}\,\delta^{\nu}{}_{[\varepsilon} x_{\rho]} \quad (9)$$

$$= g^{\varepsilon\mu}\cdot\frac{\Lambda}{3}\left(\delta^{\nu}{}_{\varepsilon} x_{\rho} - \delta^{\nu}{}_{\rho} x_{\varepsilon}\right) \quad (10)$$

$$= \left(\frac{\Lambda}{3}\right)\left(g^{\nu\mu} x_{\rho} - \delta^{\nu}{}_{\rho} x^{\mu}\right) \quad (11)$$

Second term in $\dot{\mathcal{L}}_{ds}$ is:

$$\frac{1}{2}\,\dot{T}^{\rho}{}_{\mu\nu}\,\dot{T}^{\nu\mu}{}_{\rho} \qquad (12)$$

fix factor of $\frac{1}{2}$ from anti-symmetry normalization.

$$\left(\tfrac{1}{2}\right)\cdot\left(\tfrac{\Lambda}{3}\right)^2\left(\delta^{\rho}{}_{\nu}x_{\mu}-\delta^{\rho}{}_{\mu}x_{\nu}\right)\left(g^{\nu\mu}x_{\rho}-\delta^{\nu}{}_{\rho}x^{\mu}\right) \qquad (13)$$

$$\left(\tfrac{1}{2}\right)\left(\tfrac{\Lambda}{3}\right)^2\Big(\underbrace{g^{\rho\mu}x_{\mu}x_{\rho}}_{x^2}-4x^2-\underbrace{g^{\nu\mu}x_{\nu}x_{\mu}}_{x^2}+\underbrace{\delta^{\nu}{}_{\mu}x_{\nu}x^{\mu}}_{x^2}\Big) \qquad (14)$$

$$\left(\tfrac{1}{2}\right)\left(\tfrac{\Lambda}{3}\right)^2\left(\cancel{x^2}-4x^2-\cancel{x^2}+x^2\right) \qquad (15)$$

$$=\left(\tfrac{1}{2}\right)\left(\tfrac{\Lambda}{3}\right)^2\left(-3x^2\right)=\left(-\tfrac{3}{2}\right)\left(\tfrac{\Lambda}{3}\right)^2x^2 \qquad (16)$$

The third term $\mathcal{L}_{ds}$ is:

$$-\dot{T}_{\rho\mu}{}^{\rho}\,\dot{T}^{\nu\mu}{}_{\rho\nu} \qquad (17)$$

Let's be careful with this term

first factor: $$\dot{T}_{\rho\mu}{}^{\rho}=\left(\tfrac{\Lambda}{3}\right)\delta\,\ldots \qquad (18)$$

$$\dot{T}^{\rho}{}_{\mu\rho}=\left(\tfrac{\Lambda}{3}\right)\delta^{\rho}{}_{[\rho}x_{\mu]}=\left(\tfrac{\Lambda}{3}\right)\left(\delta^{\rho}{}_{\rho}x_{\mu}-\delta^{\rho}{}_{\mu}x_{\rho}\right)\tfrac{1}{2} \qquad (19)$$

$$=\tfrac{1}{2}\left(\tfrac{\Lambda}{3}\right)\left(4x_{\mu}-x_{\mu}\right)=\left(\tfrac{\Lambda}{3}\right)\cdot 3x_{\mu}\cdot\tfrac{1}{2} \qquad (20)$$

second factor:

$$\dot{T}^{\rho\mu}{}_{\rho}=\frac{\Lambda}{3}\,\delta^{\rho}\,\ldots \qquad (21)$$

$$\dot{T}_{\rho}{}^{\mu\rho}=\left(\tfrac{\Lambda}{3}\right)\left(\delta_{\rho}{}^{[\rho}x^{\mu]}\right)$$

$$=\left(\tfrac{\Lambda}{3}\right)\cdot\tfrac{1}{2}\left(\delta_{\rho}{}^{\rho}x^{\mu}-\delta_{\rho}{}^{\mu}x^{\rho}\right) \qquad (22)$$

$$\Rightarrow \quad \dot{T}_{\rho}{}^{\mu\rho} = \left(\frac{\Lambda}{3}\right)\left(\frac{1}{2}\right)\left(4x^{\mu} - x^{\mu}\right) \qquad (23)$$

$$= \left(\frac{\Lambda}{3}\right)\left(\frac{1}{2}\right)\cdot 3x^{\mu} \qquad (24)$$

The ~~second~~ third term is then:

$$-\dot{T}_{\rho\mu}{}^{\rho}\,\dot{T}^{\nu\mu}{}_{\nu}$$

$$-\left(\frac{\Lambda}{3}\right)\cdot(3)\cdot\left(\frac{1}{2}\right)x_{\mu}\cdot\left(\frac{\Lambda}{3}\right)\left(\frac{1}{2}\right)\cdot 3\cdot x^{\mu} \qquad (25)$$

$$-\left(\frac{\Lambda}{3}\right)^2\left(\frac{1}{2}\right)^2\cdot(3)^2 x^2 = -\left(\frac{\Lambda}{3}\right)^2\left(\frac{3}{2}\right)^2 x^2 \qquad (26)$$

$$\dot{\mathcal{L}}_{dS} = \frac{h}{2k^2}\Big[$$

Double check: $\Omega = \dfrac{1}{1-\left(\sigma^2\Lambda/12\right)}$

$$\ln\Omega = \ln(1) - \ln\left(1-\frac{\sigma^2\Lambda}{12}\right)$$

$$\partial_\mu \ln\Omega = -\partial_\mu \ln\left(1-\frac{\sigma^2\Lambda}{12}\right)$$

$$= -\frac{1}{1-\frac{\sigma^2\Lambda}{12}}\,\cdot$$

$$\partial_\mu\left(1-\frac{\sigma^2\Lambda}{12}\right) = -\frac{\Lambda}{12}\,\partial_\mu\sigma^2$$

$$\partial_\mu\sigma^2 = \partial_\mu g_{\rho\sigma}x^{\rho}x^{\sigma}$$

$$=$$

$$h^a{}_\mu = \Omega \delta^a{}_\mu \qquad g_{\mu\nu} = h^a{}_\mu \eta_{ab} h^b{}_\nu$$

$$= \Omega^2 \delta^a{}_\mu \eta_{ab} \delta^b{}_\nu$$

$$= \Omega^2 \eta_{\mu\nu}$$

$$\Omega = \frac{1}{1 - \frac{\sigma^2 \Lambda}{12}}$$

$$\ln \Omega = \underbrace{\ln 1}_{0} - \ln\left(1 - \frac{\sigma^2\Lambda}{12}\right) \qquad x = \ln 1$$

$$e^x = 1$$

$$\partial_\mu \ln \Omega = -\partial_\mu \ln\left(1 - \frac{\sigma^2\Lambda}{12}\right) \qquad x = 0$$

$$= -\left(\frac{1}{1 - \frac{\sigma^2\Lambda}{12}}\right) \cdot -\frac{\Lambda}{12} \partial_\mu \sigma^2$$

$$\partial_\mu \sigma^2 = \partial_\mu \eta_{\varepsilon\sigma} x^\varepsilon x^\sigma = \eta_{\varepsilon\sigma}\left(\delta_\mu{}^\varepsilon x^\sigma + x^\varepsilon \delta_\mu{}^\sigma\right)$$

$$= \eta_{\mu\sigma} x^\sigma + x_\sigma \delta_\mu{}^\sigma$$

$$= x_\mu + x_\mu = 2x_\mu$$

x must be raised and lowered by g, not η.

$$\partial_\mu \ln \Omega = -\left(\frac{1}{1 - \frac{\sigma^2\Lambda}{12}}\right) \cdot \left(-\frac{\Lambda}{12}\right) \cdot 2x_\mu$$

$$= \left(\frac{\left(\frac{\Lambda}{6}\right) \cdot x_\mu}{1 - \sigma^2\Lambda/12}\right) = \left(\frac{\Lambda}{6}\right) \frac{x_\mu}{1 - \frac{\sigma^2\Lambda}{12}}$$

$$\Gamma^\lambda{}_{\mu\nu} = \delta^\lambda{}_\mu \partial_\nu \Omega = \delta^\lambda{}_\mu \left(\frac{\Lambda}{6}\right) \frac{x_\nu}{1 - \frac{\sigma^2\Lambda}{12}}$$

$$= \delta^\lambda{}_\mu \left(\frac{\Lambda\Omega}{6}\right) x_\nu$$

$$\boxed{\Gamma^\lambda{}_{\mu\nu} = \left(\frac{\Lambda\Omega}{6}\right) \delta^\lambda{}_\mu x_\nu}$$

The torsion

$$\dot{T}^{\lambda}{}_{\mu\nu} = \dot{\Gamma}^{\lambda}{}_{\nu\mu} - \dot{\Gamma}^{\lambda}{}_{\mu\nu}$$

$$= \delta^{\lambda}{}_{\nu}\,\partial_{\mu}\ln\Omega - \delta^{\lambda}{}_{\mu}\,\partial_{\nu}\ln\Omega$$

$$= \left(\frac{\Lambda\Omega}{6}\right)\left(\delta^{\lambda}{}_{\nu}\,x_{\mu} - \delta^{\lambda}{}_{\mu}\,x_{\nu}\right)$$

$$\dot{T}^{\lambda}{}_{\lambda\nu} = \left(\frac{\Lambda\Omega}{6}\right)\Big(\underbrace{\delta^{\lambda}{}_{\nu}\,x_{\lambda}}_{x_\nu} - \underbrace{\delta^{\lambda}{}_{\lambda}\,x_{\nu}}_{4x_\nu}\Big)$$

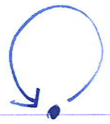
$$\dot{T}^{\lambda}{}_{\lambda\nu} = -\left(\frac{\Lambda\Omega}{2}\right)x_{\nu}$$

$$\dot{T}^{\lambda}{}_{\nu\lambda} = \left(\frac{\Lambda\Omega}{2}\right)x_{\nu}$$

$$\dot{T}_{\rho}{}^{\mu\nu} = g_{\rho\sigma}\,g^{\mu\varepsilon}\,g^{\nu\delta}\,\dot{T}^{\sigma}{}_{\varepsilon\delta}$$

$$\cancel{= \Omega^{6}\,\dot{T}}$$

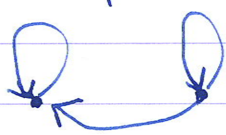
$$= \Omega^{6}\,\eta_{\rho\sigma}\,\eta^{\mu\varepsilon}\,\eta^{\nu\delta}$$

 group.

Noether: Group $\longleftrightarrow$ Conserved current

$$\begin{array}{ccc} \text{Alg. } T^A & & \int T^A \\ \text{grp. } e^{i\theta_A T^A} = g(\theta_A) & & \end{array}$$

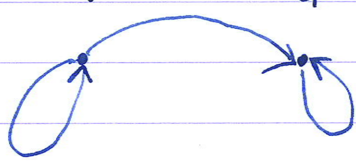
Groupoid



e.g. 2 base pts. $\Longrightarrow$ 2 identities.

So suppose we have $G = SU(2)$ with one base point, and we want to add a different group with another base point

G G'



Now, in order to have an arrow between the base pts we need to be able to multiply at least some elements of G with some of G'.

What is the nature of a groupoid?

There is a traditional and wonderfully beautiful argument that the mathematical theory of groups captures, in a parsimonious way, the conceptual structure of symmetry. The concept of symmetry that it codifies is the following: Consider a system S and a set of operations O that may be performed on the set S, an element $o \in O$ is called a symmetry of S if observations of S done just before and just after the performance of o are indistinguishable.

With this philosophical conception of system and symmetry operation, one is led parsimoniously to the abstract notion of group as it was defined by Galois.

I am looking for the analogous philosophic construction that would lead to the notion of groupoid. I have always felt uncomfortable with things like: Hopf algebra symmetries, because of the beauty and generality of the above argument for the parsimonious connexion between symmetry and the abstract theory of groups. As of right now (Tuesday September 4th 2012) it seems to me that, if we simply assume that the system S has an internal structure we may be led directly to the groupoid structure ~~with~~ in a parsimonious fashion.

So, let's try to prove this:

Let $S = S_1 \cup S_2 \cup S_3$ for example.

Now we will look at the space of all operations O as we did in the case of groups.

Sept. 10 2012

Energy and Teleparallel Gravity

Some scratch on Schwarzschild

naieve choice of tetrad

Spherical symmetry $\rightarrow (t, r, \theta, \varphi)$

Susskind Lecture

Big problems with global multiverse.

Foundations of quantum mechanics

Bousso & Susskind

Foundations of Quantum Mechanics $\longleftrightarrow$ Foundations of Cosmology.

Issues:

1. At what pt. do the virtual realities described by the wave-fn. become objective realities
2. What is the meaning of probability
3. Who or what gets to observe the whole multiverse.

3 fictions

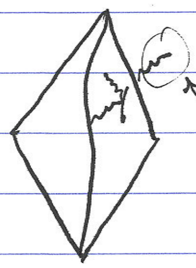
1. Wfn. collapse ~~need not~~ is an irreversible process
2. $\text{Prob.}(A) = \lim_{N \to \infty} \frac{N_A}{N_{TOTAL}}$ the fiction is that N can go to infinity.
3. There is a multiversal archivist. Someone who observes the multiverse.

<u>Decoherence</u> - modern form of wfn collapse. wfn gets entangled with the environment.

Postulate 1:

Things happen. There exists a mechanism for irreversible decoherence.

(This rules out <u>closed</u> <u>cosmologies</u>. Quantum recurrences.) because Quantum recurrences undo decoherence.
dS is an escape
AdS is a better closed box cosmology.



← cannot recohere.

$\Rightarrow$ <u>event</u> <u>horizons</u> may allow real decoherence.

Note dS decays in a time <u>much</u> smaller than the recurrence time.

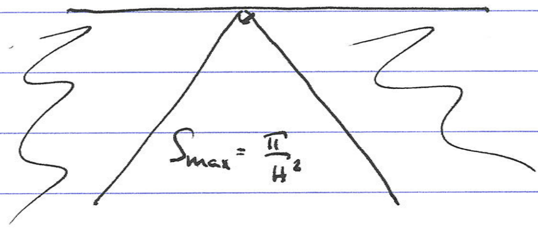
Postulate 2:

frequentist $\quad P(A) = \lim_{N \to \infty} \frac{N_A}{N_{tot}}$

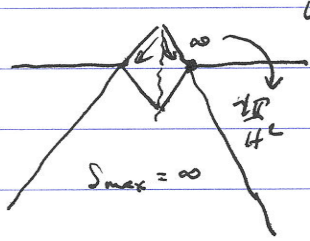
Need
a) ∞ # of trial
b) way to order them so we don't get $\frac{\infty}{\infty}$.
ordering or regulating.

Postulate 3: There really is a multiversal archivist. The mathematical observables are physical observables.

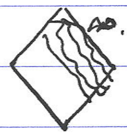
multiversal archivist → subsystem. Must be able to record everything in the system in which it is contained.



Imagine a very late decay of dS to bubbles with zero c.c. (Banks?)



why does this hat have an ∞ amount of information in it.



$\{x^0, x^1\}$ / Algebra $f(x^0, x^1)$

$[x^0, x^1] = f(x^0, x^1)$ Representations.

$$f(x^0, x^1) = A + Bx^0 + Cx^1 + Dx^0x^1 + Ex^1x^0 + \dots$$

$$\text{or} \sim Ae^{-Bx^1}$$

$$e^{-Bx^1} \simeq 1 - Bx^1$$

$$[x^0, x^1] \simeq A(1 - Bx^1)$$

$$[x^0, x^1] \simeq A - (AB)x^1$$

$$x^0 | x^0 \rangle = x^0 | x^0 \rangle \qquad \langle x^1 | \psi \rangle = \psi(x^1)$$

$$x^1 | x^1 \rangle = x^1 | x^1 \rangle$$

$$\cancel{(x^0 + ix^1) | 0 \rangle = (x^0 + ix^1)} \qquad \langle x^0, x^1 \rangle$$

Why isn't the wfn a fn of space and time?