Physics notebook volume 17: 11/05/09 –04/22/10

Summary and context

This is a continuation of the gauge gravity project. The folder contains the main realizations and by the end of it there are just some technical calculations to do in order to determine the Wong and Lorentz equations for a quantum mechanical particle moving in an external gravitational field. The work as usual is often discussed with Sergio and completed in the winter and spring of 2009/2010.

Table of Contents

1. Proof of the relation between the field strength and the gauge field that Tanimura failed to reproduce in [17]. I did this on page 1.
2. On page 3 I prove that the Wong equation assumed in [17] is actually derivable from the ***Anzatse***

$$\dot{I}^a = H^a(x, \dot{x}, I)\,, \tag{1.1}$$

the assumption of linearity of H^a in I ,and the assumption that it can be factored as a function of x times a function of $\dot{x}$.

3. The question: "Do the differential conservation law and the Bianchi identities determine the field equations in generally covariant theories?" is asked.
4. ***Lorentz group in the adjoint representation***

$$[S_{ab}, S_{cd}] = F_{abcd}{}^{ik} S_{ik} \tag{1.2}$$

where

$$F_{abcd}{}^{ik} = (\eta_{bc}S_{ad} + \eta_{ad}S_{bc} - \eta_{bd}S_{ac} - \eta_{ac}S_{bd})^{ik} \tag{1.3}$$

and $(S_{ab})^{ik}$ are the 4 x 4 matrix representation of the Lorentz generators to be found e.g. explicitly in [20].

5. Proof of the ***uniqueness*** of solutions to the isospin Wong equation on page 25 of Novikov, Vol. 2.
6. Page 5 has a proof of

$$\dot{I}^a = \frac{m}{i\hbar} A_\mu{}^a{}_b(x)\dot{x}^\mu I^b \tag{1.4}$$

7. The Wong equation for the S^{ab} on pages 6 and 7.
8. Some notes on Lorentz force, page 7.
9. ***Jacobi identity*** for the Lorentz generators, page 7.
10. A ***new representation*** for Lie algebras on page 8.

11. ***Relation between the gauge field and field strength*** for the Lorentz generators, pages 8-9.
12. The Lorentz force for gauge gravity with only the Lorentz group is found to be linear and not contain the essential features of gravity and, so, should not be called *gauge gravity*, just gauge theory for the Lorentz curve.
13. There's a conflict between covariance and energy boundedness, i.e. the fact that

$$[x^{\mu}, p^{\nu}] = -i\hbar \tag{1.5}$$

14. First attempts to try to get a nonlinear velocity factor in the force by introducing the Poincare algebra, page 10. I thought it would either arise from relaxing the requirement of linearity in J^{ab}, or by introducing the tetrads which can change Latin to Greek indices—it was the latter that worked and connected up with heretofore studied approaches e.g. that in [10]
15. Ordinary direct product considered with two different Yang Mills groups generated by I^a, J^a
16. Between pages 11 and 15, I was worried about how to get a force law quadratic in the velocities. On page 15 this was solved. I realized that the Wong equation for the momentum is quadratic in the velocities.
17. What was on page 15 was not quite right in terms of its treatment of the potentials. This problem was solved later with an introduction to tetrads and local translation invariance.
18. More details on adjoint representations of the Lorentz algebra

$$(S_A{}^{cd})^{ab}{}_{ik} := f^{abcd}{}_{ik} = -i(\eta^{ac}\delta^d{}_i\delta^b{}_k - \eta^{bc}\delta^d{}_i\delta^a{}_k + \eta^{ad}\delta^c{}_i\delta^b{}_k - \eta^{bd}\delta^{ci}{}_i\delta^a{}_k) \tag{1.6}$$

19. November 22: Try assuming parallel transport for both S^{ab}, P^a and solve simultaneously for both field strengths. When this was done, it yielded a contradiction as the gauge potentials were not correctly introduced because of the difficulty in introducing local translations.
20. On January 11, page 40, I figured out how to introduce local translations correctly, relying on the insights in [10]. **Page 40 contains the main insight that makes the gauge gravity paper possible. The key is to introduce a compensating field for local translations.**
21. Thought about comparing the approach in [10] to the approach to gravity using supersymmetry.
22. More analysis of direct products on page 41.
23. Page 43 comparison of the translation covariant derivative with the covariant coordinates in noncommutative field theory is introduced in [9].
24. Three important angles:
    a. Supersymmetry in the Feynman formalism and the Wong equations for supersymmetry
    b. The connexion to noncommutative theories
    c. The connexion with Clifford algebras, especially regarding the reformulation of GR in [3].
25. A paradox involving strong noncommutativity is considered on page 44.
26. The species problem as a quantum gravity problem.
27. Page 45, wondering what is at the root of the connexion between nonholonomicity and local translations
28. Page 45, starting to consider the Wong equation origin for the translation gauge potential.
29. Wondering how anholonomic bases arise in the noncommutative context.

30. Page 48, distinguishing between the two types of momenta: the tetrads and the particle momenta.
31. Local translations versus diffeomorphisms, bottom of page 50.
32. List of topics:
    a. Nonlinear realizations
    b. Coherent states and Minkowski limit
    c. Coherent states and nonlinear realizations
    d. Anholonomic fluctuations
    e. Anholonomic quantization, constraints versus change of variables
    f. Locality and noncommutativity
    g. Supersymmetry and stability
    h. The Wong limit and particles.
33. Wong equations expressed in Weinberg §5.1.7 and §5.1.8 suggest that the gauge potential appearing in the Wong equations for both the Momentum and Lorentz generators is the Lorentz gauge potential $A_{\mu}{}^{ab}{}_{cd}$.
34. Page 53, January 24, Goldstone theory of coordinates
35. Wong equations from Weinberg
36. Checking lots of calculations from Weinberg [22] (e.g. §5.1.7 , §5.1.8, §12.5.17)
37. Reviewed local Lorentz invariance and general covariance in [22]
38. Page 57, thesis is proposed that when quantum corrections are added to the correct gauge formulation of gravity, nonlocality will be generated; in other words, noncommutativity is already present in some sense in ordinary classical gravity.
39. Mohrbach papers cannot be trusted. [4], [15]
40. Connexion between noncommutativity and the gauge formulation of gravity is explored further
41. Three comments:
    a. There is a very deep connexion between quantum corrections and noncommutative ones, especially in Q.M. (0+1 field theory)(and possibly in 0-dimensions). However, the noncommutativity is in a *field space*.
    b. When the gauge structure of GR is elucidated, especially with respect to translations, an addition subtlety appears regarding which objects are coordinates like x^{μ} and which are more like fields ξ^{a}.
    c. Gauge theory is already deeply connected with quantum mechanics, since the only unitary quantum theories of spin-1 particles are gauge theories.
42. Pages 58 and 59 are my own attempt to get the results claimed in [4] and [15].
43. ***Diagrams*** of the physics of Feynman's proof are drawn.
44. Mathematical diagram connecting gauge structure, noncommutativity, and quantum mechanics
45. A big list of threads (citations have to be put here!!):
    a. Nonlinear realizations [21] § 19.6
    b. Nonholonomic manifolds and quantization [13] [14] [12]
    c. Jet bundles [1?]
    d. Deformation quantization
    e. Clifford algebras and gauge theories

    f. Noncommutativity
    g. All the Maxwell equations from S-duality
    h. Supersymmetry and the connexion between supersymmetry and noncommutativity
    i. Uncertainty relations from the various commutators
    j. Noncommutativity's connexion with nonholonomicity
    k. Noncommutativity's connexion with generalized gauge structures in the Chinese book
    l. The problem of motion in gravity theories and gauge theories
    m. Constraints and $[x^{\mu}, p^{\nu}] \square \eta^{\mu\nu}$.
    n. C.f. string theory and 't Hooft's comments.
    o. The possible necessity of supersymmetry in the Lagrangian description of Wong's equations
    p. S-duality for non-Abelian gauge theories
    q. $U(1)\times\tilde{U}(1)$ in QED
    r. Kibble versus Hehl arguments about the arising of the tetrad as a gauge field
    s. Clifford analysis and superspace and hep-th/0707.2859
    t. Teleparallel gravity in noncommutative gauge theory
    u. Equivalence principle violation and its implications for constance and gauge gravity
    v. Deformation quantization (Fedosov→differential geometry)
    w. The Wong limit and its meaning in
        i. Gauge theory
        ii. Gauge gravity
    x. Torsion and topology – See the paper "The torsion formulation of gravity" written in 2009
    y. Pseudo-Lie groups
    z. Making the tangent space affine and affine differential geometry generally and its relation to composite fiber bundles

46. Gilmore, pages 66-70—excellent discussion of nonlinear realizations in 1-dimension
47. Fundamentally, nonlinear realizations are required due to the fact that "local Lorentz coordinates" ξ^{a} are elements of G/H *and* are acted upon by G. So the group plays a dual role:
    a. Topologically parameterizing the "tangent space" ξ^{a} .
    b. Defining the symmetries of this very same tangent space and acting upon it.
48. Can the fact that GR →EOM for a test particle be understood from the entropic force picture? (cite Verlinde)
49. What is the connexion between the gauge theory of translations and the entropic force picture of gravity?
50. How can the gauge peculiarities of gravity explain the statistical interpretation of gravity?
51. The Wong equations and local translations, pages 61-62. Covariant Lie derivatives, Appendix C of [11].
52. A comparison of [19], [18] and [9]

53. One possibility is that gauge gravity plus quantum corrections equals noncommutativity of coordinates.
54. There may be too many translational degrees of freedom in the commutative theory of gravity to render it quantum mechanically well-defined, page 63.
55. Starting to consider the Seiberg-Witten map; trying to get from "translations as gauge transformations"→$[x^i, x^j] = i\theta^{ij}(x)$
56. How does the restriction to $U(N)$ in noncommutative gauge theory square with the fact that noncommutative theories contain *translations* as gauge transformations?
57. The canonical transformations that correspond to translations are written down, and the Seiberg-Witten map for translations is examined.
58. An idea about time (Anthony asks, "How do you know it's 2010?"). To define a time interval, we need at least
    a. A periodic system
    b. Enough space to accommodate the number of bits that are encoded in the number of oscillations of the periodic system contained in the time interval we wish to measure.

    This seems to lead to an uncertainty in the time measurement

$$\Delta t = Te^{-\frac{A}{l_p^2}} \tag{1.7}$$

59. In order to find the Seiberg-Witten map from noncommutative translations, we have to calculate

$$\{J^{ab}, P^c\} = ? \tag{1.8}$$

    This is on pages 66-68, where I sketched a method for figuring it out.
60. Fermi transport equation is probably the particle limit of the Riemann-Cartan equation of motion for a pole particle.
61. March 5—I decided to put nonlocality aside and calculate
    a. The Wong equations
    b. The Lorentz force

    I expected to find something like the Gasperini equation in [8] (10.90)

62. Attempts to write the Lorentz equation for gravity as in, for example, [2]
63. In the original analysis of Tanimura [17]

$$\eta^{uv} \to g^{uv}(x) \tag{1.9}$$

    a. The commutation relations are deformed as above
    b. The momentum is kept holonomic
    c. The result is the unique torsion-free Christoffel connexion
64. The formula

$$[P^{a,}, P^b] = T^{ab}{}_c(x) P^c \tag{1.10}$$

where T is the torsion

65. The connexion with the Chinese book [6] is considered. The theory has a chance to be noncommutative if and only if $\overset{T}{A}$ is nonholonomic. I.e.

$$\overset{T}{A}{}^{b} P_{b} \tag{1.11}$$

satisfies

$$[\overset{T}{A}, \overset{T}{A}{}'] \neq 0 . \tag{1.12}$$

Some displacements must be gauge transformations for noncommutativity to manifest , and this seems to be the case here.

66. Page 73, the wave function is considered.
67. An interesting form of the momentum algebra can be considered. In the notation used in [2], we find

$$[P_{\mu}, P_{\nu}] = ic^{2} \overset{T}{F}_{\mu\nu} \tag{1.13}$$

Or equivalently,

$$[P_{\mu}, P_{\nu}] = -iT^{\rho}{}_{\mu\nu} P_{\rho} \tag{1.14}$$

In the form (1.13), it appears that the usual commutation relations for momentum are modified by some space-time dependent electromagnetic-like field. In the form (1.14) we have a family of Lie algebras—one at each point in spacetime. Perhaps we can quantize and explore the analog of the *lowest Landau level, which might induce space-time coordinate noncommutativity*.

68. At this point in time, I thought that the sticking point for the Wong equations would be whether or not to adopt the Pereira formalism, which is on a shaky mathematical footing, or the Tresguerres formalism in [19] and [18]. The latter is much clearer mathematically, and the former much clearer physically. I wanted to do both and check for consistency, but it turned out the Brazilians' final formula worked formally, and I didn't carefully consider Tresguerres' formalism in this notebook.
69. Are we sure that $\overset{T}{A}$ depends only upon x and not upon $\dot{x}$? Yes.
70. The interaction term for the Wong equations is provided in [16].
71. Remember to put in caveats about the compactness of the Poincaré, respectively Lorentz group.
72. Why does equation 16 of [7] work if the Wong equations for Poincaré are given by the action in [16]?
73. We have two momenta

a. p^{μ} →the ***holonomic***, ***worldline indexed***, ***particle***, ***momentum***

b. $P^{a} = \delta^{a}{}_{\mu} p^{\mu} + m \overset{T}{A}{}^{a}$, which comes from $e^{a} = \delta^{a}{}_{\mu} \dot{x}^{\mu} + \overset{T}{A}{}^{a}(x(\tau))$ is the ***nonholonomic, gauge (local translation) invariant***, ***tangent space indexed,*** momentum.

74. Page 76, calculated the variation of the determinant of the metric for Zach.
75. On March 28, page 77, I began to work on getting the equation of motion in [7].

$$h^{\alpha}{}_{\mu} \frac{du_{\alpha}}{ds} = -c^{2} F^{\alpha}{}_{\mu\nu} u_{\alpha} u^{\nu} \tag{1.15}$$

$$h^{\alpha}{}_{\mu} = \partial_{\mu} x^{\alpha} + c^{-2} B^{\alpha}{}_{\mu} =: \mathsf{D}_{\mu} x^{a} \tag{1.16}$$

76. Proof that the gauge field

$$A^{\mu}{}_{\alpha}(x, \dot{x}, P) := [x^{\mu}, \dot{P}_{\alpha}] \tag{1.17}$$

77. A subtlety: the $\overset{T}{A}{}^{a}$ on page 40, equation 8, is

$$\overset{T}{A}{}^{a}(x) = \overset{T}{A}{}^{a}{}_{i}\, dx^{i} = \overset{T}{A}{}^{a}{}_{i}\, \dot{x}^{i} d\tau \tag{1.18}$$

So $\overset{T}{A}{}^{a}(x)$ implicitly depends on the momentum *because it is a 1-form*. This is the connexion between Pereira and Hehl, I think.

78. I don't know what the hell Pereira thinks he means by

$$\frac{\partial x^{\alpha}}{\partial x^{\mu}} \tag{1.19}$$

79. The inconsistency between Pereira [2], on the one side, and Hehl [10] and Blago [5], on the other

a. Pereira: $h^{a}{}_{\mu} = \partial_{\mu} x^{a} + \overset{T}{A}{}^{a}{}_{\mu}$

b. Hehl, Blago: $h^{a}{}_{\mu} = \delta^{a}{}_{\mu} + \overset{T}{A}{}^{a}{}_{\mu}$

80. Translation covariant derivative defined on page 80
81. Comparison of Tresguerres approach to the Lorentz type equation for gravitation [19] with the Pereira approach [7]
82. On April 1, I made the decision to ignore all the difficult issues and work formally aiming at the Wong equations for translation, namely

$$h_{\mu}{}^{\alpha} \frac{du_{\alpha}}{ds} = F^{\alpha}{}_{\mu\nu} u_{\alpha} u^{\nu} \tag{1.20}$$

or

$$\frac{du_\beta}{ds} = h_\beta{}^\mu T^\rho{}_{\mu\nu} u_\rho u^\nu \tag{1.21}$$

83. On page 83, I consider the lowest Landau levels in the non-Abelian case. This may allow the presence of noncommutativity to manifest itself within the teleparallel approach.
84. On April 2, I did another computation of the Wong equations starting from the bracket between space-time and the generalized momentum.
85. On April 3, there is an analysis of the dimensionality of the fundamental brackets in the presence of a magnetic field and a noncommutativity parameter. There is a suggestion from S. Ghosh that the noncommutativity parameter is associated with spin generators, which suggests that the coordinates may remain commutative until the Lorentz group is added to our translation gauge theory. Alternatively, something like torsion could appear, which would only require local translations. Possibly in the light of SW map, commutative and noncommutative space-times may be gauge equivalent. I also notice here that the fundamental brackets used to get the Wong equations have a problematic asymmetry in that the gauge invariant momenta appear bracketed with the **gauge dependent** coordinates. It seems natural to replace these latter by gauge invariant coordinates.

$$\begin{aligned}
&p_x = o_x + \xi^a e_a \\
&\{x^\mu, x^\nu\} = \theta = scalar \\
&[x^\mu, p^\nu] = -i\hbar g^{\mu\nu} \\
&w^{\mu\nu}
\end{aligned} \tag{1.22}$$

86. Idea: Gauge gravity + quantum theory = noncommutativity. Page 86. The idea would be to use the gauge invariant position and affine quantization in ordinary Galilean space.
87. April 5, the SW map for fluids is explored by Jackiw. It would be interesting to consider the analogous SW map in the theory of elasticity.
88. On April 6, I went through the computations of Suskind and Bigatti on the physical origins of the momentum shift.
89. On April 8, I was thinking about composite fiber bundles. More computations on the Wong equations.
90. Computations on the Jacobi identity.
91. Quantum particle in a classical background field: typical phenomena
    1. Pair creation
    2. Monopoles and solitons
    3. Duality
    4. Sensitivity to topology of configuration space
    5. Hints of supersymmetry in the Wong equations
    6. Dirac brackets and the formation of commutators

7. Uncertainty principle: gravitational analog of Bohr argument in electrodynamic theory

92. Lie bracket versus Poisson bracket. A consistency condition: Lie-Poisson manifold
93. On April 20, can we maintain Lorentz invariance and introduce noncommutativity by imposing

$$\{x^{\mu}, x^{\nu}\}\{x_{\mu}, x_{\nu}\} = \theta^2 = scalar \tag{1.23}$$

Doesn't Yoneya do something like this?

94. Literature on composite fiber bundles, Nash and Sen and Baez
95. Generally for the question of noncommutative space-time, the first versus second quantization is quite significant. In the first quantization scheme the coordinates are dynamical quantum operators, and their algebras can therefore be deformed without immediately risking Lorentz violations.
96. A few computations on Wong equations on the last page, 96.

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