TASI 2005

6/8/05

From Brane Inflation: From Superstrings to Cosmic Strings.

Henry Tye Cornell

- Cosmology
- Brane Inflation - Toy
- Cosmic Strings
- Detection / Test
- KKLMMT model → KKLT model + $D3\ \overline{D3}$
- More on Cosmic Strings

Cosmology:

RW sol'n

$a \sim t^{2/3}$ matter dominated $\rho \sim a^{-3}$ $P = 0$

$a \sim t^{1/2}$ Radiation $\rho \sim a^{-4}$ $P = \frac{\rho}{3}$

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$$

$$\left(\frac{\ddot{a}}{a}\right) = -q = \frac{-4\pi G}{3}(\rho + 3P)$$

Negative Pressure

$P \geq 0 \Rightarrow \ddot{a} < 0$

$P < -\frac{\rho}{3} \Rightarrow \ddot{a} > 0$

$$\frac{\rho}{\rho_c} - 1 = \Omega - 1 \qquad \frac{k}{H^2 a^2} = \frac{8\pi G\rho}{3H^2} - 1$$

$$k = \underset{\text{open}}{\overset{\text{closed}}{\pm}} 1, \underset{\text{flat}}{0}$$

$\Omega_0 \sim 1 \pm 0.03$

$$\Omega_0 - 1 \sim \begin{cases} kt & \text{r.d.} \\ kt^{2/3} & \text{m.d.} \end{cases}$$

Inflaton

$$\Delta\phi = \gamma'$$

Horizon 'Problem'?

- Gravitational soln
or - Effective field thy. soln ?

Brane antibrane force (Banks, Susskind)
as inflaton

Gukov: $N=1$

Motivation

i) Realistic Models P.P.

ii) Techniques in S.T.

iii) Math

$$\begin{tikzcd} \text{String Theory } (D=10,11,12,\ldots) \arrow[d, squiggly] \\ \text{4D field thy.} \end{tikzcd} \qquad \begin{tikzcd} M^4 \times \underline{X} \\ \text{compact} \arrow[u, squiggly] \end{tikzcd}$$

Geometry of X $\Leftrightarrow$ Physics of M^4

What characteristics of L emerge from string compactifications.

$$\int d^4\theta \, \bar{\Phi} \Phi \rightsquigarrow L = (\partial_\mu \Phi)^2 + \psi \not{\partial} \psi$$

$$L = G_{i\bar{j}}(\Phi) \left[(\partial_\mu \phi^i)(\partial^\mu \bar{\Phi}^j) + \psi_i \not{\partial} \psi_j \right]$$ (σ-model)

$\underline{N=1}$: $$G_{i\bar{j}} = \frac{\partial^2 K(\Phi, \bar{\Phi})}{\partial \Phi^i \partial \bar{\Phi}^j}$$ Kahler metric in space of fields. (see Wess & Bagger)

$$\int d^4\theta \, K(\Phi, \bar{\Phi})$$

~~~

M-theory compactification on smooth G_2 manifolds

size$(X) >> l_p$
(but less than experimental scale)

$$S_{11} = \int d^{11}x \left(\sqrt{-g} R - \frac{1}{2} G \wedge * G - \frac{1}{2} G \wedge G \wedge C - \ldots \text{fermions} \right).$$

G 4-form $G = dC$ (C - 3 form)

$$g_{\mu\nu} = \langle g_{\mu\nu} \rangle + \delta g$$

$$G = \underset{\substack{\| \\ \text{background flux} \\ \text{or } G\text{-flux}}}{\langle G \rangle} + \delta G$$

δG ← $d(\delta C)$

first consider $G_{flux} = 0$

Equations of motion:

$$R_{ij} = 0$$

$$\overset{\text{background flux zero here.}}{\mathbb{R}^4} \times \underline{X}$$

$$d^* G = \frac{1}{2} \underbrace{G \wedge G}_{\substack{8\text{-form} \\ \text{on a 7 manifold}}} = 0 \longrightarrow d * dC = 0 \longleftrightarrow \Delta C = 0$$
~~~

δg : Ricci Flat deformations of X. $\longleftrightarrow$ massless scalar fields in 4 dimensions. (MODULI).

$\boxed{\mathrm{Def}(X) = ?}$ can figure it out.

Focus on C-fields first :

$\mathcal{C}$

$$\Delta \delta C = 0 - (\Delta_4 + \Delta_7)\delta C = 0$$

$$\delta C = \sum_i{}' \phi_i^{(x)} \omega_i^{(3)}(y) + \sum_j{}' A_j \wedge \omega_j^{(2)}$$ Ansatz

$\llcorner$ basis of harmonic 3 forms

(no harmonic 1 form)

$\omega_i^{(3)} \in H^3(X)$

$\omega_i^{(2)} \in H^2(X)$

$G_{0123} = 0$?
constant energy density.

$\Rightarrow$ b_3 real scalars (completed to b_3 chiral multiplets from δg)
b_2 vector multiplets

$\Rightarrow$ $\dim \mathrm{Def}(X) = b_3$

$$\int_X G \wedge * G \rightsquigarrow G_{ij}(\partial\phi_i)(\partial\phi_j)$$

$$G_{ij} = \frac{1}{\mathrm{vol}(X)} \int_X \omega_i^{(3)} \wedge *_7 \omega_j^{(3)}$$

Conclusion :

* Abelian gauge fields ☹
* No charged matter ☹ (massless moduli)

Turning on fluxes:

$$G_{flux} \neq 0 \quad \in H^4(X)$$

$$S = \int G \wedge * G + \dots$$

in $N=1$ thy: • $V \sim (flux)^2$

• $V \sim |D_{\varphi_i} W|^2$

$\Rightarrow$ W is linear in flux. $\boxed{W \sim flux}$

$$\delta_{susy} = 0 \iff D_{\varphi_i} W = 0$$

Heuristic arguement for W:

Invariant forms: X general special Holonomy manifold.

$\nabla \xi = 0$ Killing spinor $\quad \underset{\uparrow}{\omega^{(p)}} = \xi^{+} \Gamma_{i_1 \cdots i_p} \xi$

covariantly constant, invariant under $Hol(X)$.

($\omega^{(p)}$'s can also characterize special Holonomy manifolds)

* S_i minimal (supersymmetric) cycle, $\dim S' = p$

$$Vol(S) = \int_{S'} \omega^{(p)} \qquad // \text{ usually } Vol(S) = \int_S \sqrt{-g}$$

geometry is encoded in $\omega^{(p)}$'s rather than metric!

Kahler 2 form.

Hol	Invariant p-form	Susy p-cycle
	$p=3 \to \Omega$	h.l cycles
SU(3)	p even; $\frac{1}{n!} K^n$	Special Lagrangian Cycles.
G_2	$p=3 \quad \Phi$	Associative
	$p=4 \quad *\Phi$	co-Associative
Spin(7)	$p=4 \quad \Psi$	Cayley form

$\omega^{(p)}$'s characterize geometry

Thm: $Hol(\underline{X}) = G_2 \Longleftrightarrow \begin{cases} d\Phi = 0 \\ d*\Phi = 0 \end{cases}$

Pf: $ds^2 = \sum_{i=1}^{7} e_i \otimes e_i$

$\Phi = \frac{1}{3!} \psi_{ijk} e^i \wedge e^j \wedge e^k$ satisfies $\begin{matrix} d\Phi = 0 \\ \delta\Phi = 0 \end{matrix}$

$g_{\mu\nu} = g(\Phi)$

Part of theory of Calibrated Geometries:

Harvey & Lawson '82

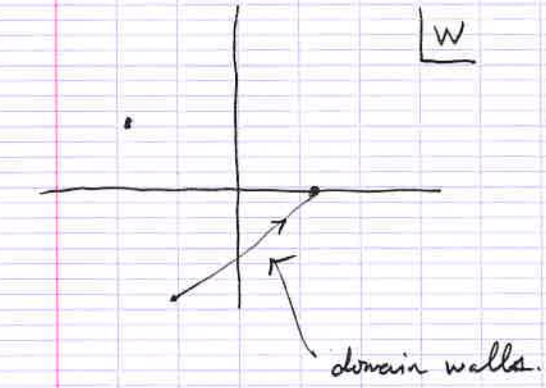
Def: a closed p-form ψ, $d\psi = 0$ is called calibration if for $\forall$ p-dim'l $S \subset \underline{X}$ and every $x \in S$

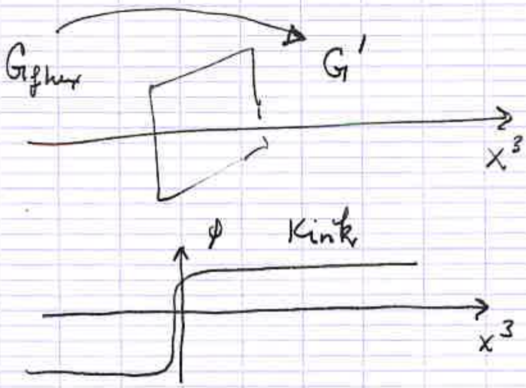
$\psi\Big|_{T_x S} \leq vol(T_x S)$ (ψ is a multiple of vol form on S)

$=$

$DW = 0$

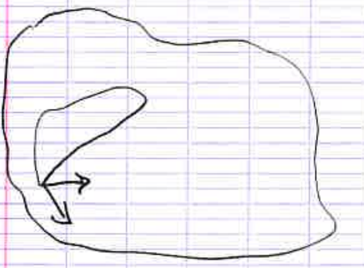
(also we are back to G_2).





$N=1$ SUSY in $M^4 \longleftrightarrow$ Special Hol. of $\overline{X}$.

$(\overline{X}, g)$



$\nabla \longleftrightarrow g$ dim $|X| = m$.

$SO(n)$

$$\begin{array}{c} Hol(\overline{X}) \\ \shortparallel \\ Hol(g,\overline{X}) \end{array} \leq \underbrace{SO(n)}_{\text{general case}}$$

Special Cases: $(Hol(X) = SO(n))$ Berger (1955)

Metric	Holonomy	Dimension
Kähler	$U(\frac{n}{2})$	n even
Calabi-Yau	$SU(\frac{n}{2})$	n even
Hyper-Kahler	$Sp(\frac{n}{4})$	multiple of 4.
Quaternionic	$Sp(\frac{n}{4})/Sp(1)$	n = mult of 4
/ —	G_2	7
/ —	Spin(7)	8
/ —	Spin(9)	16

locally symmetric

max.

↑

locally orbit of a group.

$$G_2 \cong \mathrm{Aut}(\overset{\overset{\text{octonions}}{\downarrow}}{\mathbb{O}}) \quad , \quad G_2 \subset GL(7,\mathbb{R})$$

$$x^i \in \mathbb{R}^7$$

s.t. preserves:

$$\Phi = \frac{1}{3!}\, \underset{\underset{\pm 1,\, 0}{\uparrow}}{\psi^{ijk}}\, dx^i \wedge dx^j \wedge dx^k \qquad \sigma_i \sigma_j = -\delta_{ij} + \psi_{ijk}\sigma_k$$

$$i,j,k = 1, \dots, 7$$

<u>Ex</u> $X = S^n$

$$Hol(S^n) = SO(n)$$

Ex $X = T^2 = \mathbb{R}^2 / \mathbb{Z}\oplus\mathbb{Z}$

$$ds^2 = dx^2 + dy^2 = dz\, d\bar{z}$$

$$\Rightarrow \quad Hol(T^2) = \{e\}$$

$$\Rightarrow \quad Hol(T^n) = \{e\}$$

<u>Exc</u>: $ds^2 = \dfrac{|dz|^2}{1+|z|^2}^2$

$$Hol = ?$$

Diffeos or metric preserving diffeos?

$$M^4 \times \underbrace{X}_{\Downarrow\ \mathcal{N}=1 \text{ susy.}}$$

$R_{ij} = 0$

$\underset{SUSY}{\delta\Psi_{r}} = \boxed{\nabla\xi = 0}$ Killing spinor eqn. $\Rightarrow$ ξ invariant under parallel transport

$\Rightarrow$ Hol is s.t. spinor rep. contains a singlet ($\Rightarrow$ Hol $\neq SO(n)$)

e.g. $Hol(X) = G_2 \subset SO(7)$ $8 = 7 \oplus 1$

$R_{ij}=0 \Leftarrow \{ X$ = Calabi-Yau, Hyper Kahler, Exceptional G_2, or Exceptional Spin(7)

Kahler $\rightarrow R_{ij} \neq 0$ $g_{i\bar{j}} = \frac{\partial^2 K(z,\bar{z})}{\partial z^i \partial \bar{z}^j}$

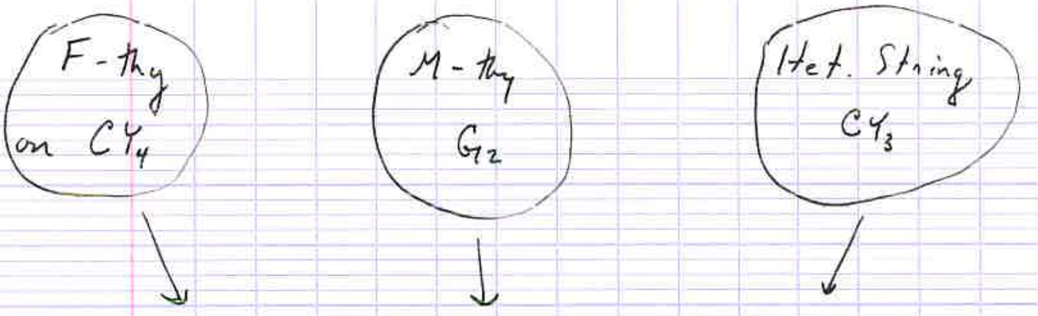
Manifold X	T^n	CY_3	X_{G2}	$X_{Spin(7)}$
$\dim_{\mathbb{R}}(X)$	n	6	7	8
Hol (X)	$\{e\} \subset$	$SU(3) \subset$	$G_2 \subset$	Spin(7)
raction SUSY preserved	$1 >$	$1/4 >$ ($N=2$)	$1/8 >$ ($N=1$)	$1/16$

Conclusion:

Larger the holonomy group, the ~~more~~ less SUSY is preserved.

Hol ~ measure of symmetry of underlying manifold.

F-thy	12
M-thy	11
Het String	10



$\longrightarrow$ 4D minimal low energy effective QFT

Notice $Hol(X \times T^4) = Hol(X)$

$de^2(X^1 \times X^2) = de^2(X^1) + de^2(X^2)$ – reducible.

?Criterion: $X \rightsquigarrow$? reducible.

Intermezzo (Basic Topology)

How do we distinguish different $\mathfrak{X}$?

* dim, Hol, Betti #'s, (co)homology groups

$H^p(X) = {}^{\text{closed}}/_{\text{exact}}$

$H_p(X) = {}^{\overset{\text{(p-cycles)}}{\text{closed}}}/_{\text{boundary}}$

$= {}^{Z_p}/_{\delta Z_p}$

$b_p = \dim H_p \qquad b_p = b_{n-p}$ (smooth cpt.)

$\mathfrak{X}$ cplx manifold, $\partial, \bar\partial$

$\partial^2 = \bar\partial^2 = 0 \longrightarrow H^{p,q}(X)$

$H^k(X) = \bigoplus_{p+q=k} H^{p,q}(X)$ Hodge decomposition

$h^{p,q} = \dim H^{p,q}$ ← Hodge numbers. $h^{p,q} = h^{n-p,n-q}$

Ex: $X = T^2$ $b_i = \begin{cases} 1 & i=2 \\ 2 & i=1 \\ 1 & i=0 \end{cases}$ $h^{pq} = \begin{array}{ccc} & 1 & \\ 1 & & 1 \\ & 1 & \end{array}$ hodge diamond.

$p \nwarrow \quad \nearrow q$

$b_1(T^n) \neq 0$ not simply connected.

$$\boxed{\text{Criterion irreducible Hol} \rightsquigarrow b_1(X) = 0}$$

Exc: a) simple example of reducible holonomy manifold which doesn't satisfy above. ($b_1(X)$).

b) simplest Ricci flat ——"—— (minimal dimension).

c) improve criterion using some other hodge numbers.

X Calabi-Yau

$\dim_{\mathbb{C}} X = 2$ $X = K3$ $h^{i,j} = \begin{array}{ccccc} & & 1 & & \\ & 0 & & 0 & \\ 1 & & 20 & & 1 \\ & 0 & & 0 & \\ & & 1 & & \end{array}$ ← $b_1 = 0$

$\dim_{\mathbb{C}} X = 3$ $X =$ $h^{ij} = \begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & h^{22} & & 0 & \\ 1 & & h^{31} & & h^{12} & & 1 \\ & 0 & & h^{11} & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array}$ ← $b_1 = 0$

Exc: How many Hodge numbers should you tell your friend for CY_4, CY_5?

X G_2 manifold ($\dim_{\mathbb{R}} X = 7$)

$$\begin{array}{cccccccc} b_0 & b_1 & b_2 & b_3 & b_4 & b_5 & b_6 & b_7 \\ \| & \| & ? & ? & \| & \| & \| & \| \\ 1 & 0 & & & b_3 & b_2 & 0 & 1 \end{array}$$

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Refs:

hep/th - 040919 Review w/Acharya
hep-th/ 9906070 } today
hep-th / 9911011

Low Energy Effective action:

$$L = L_{gauge} + L_{matter} \underset{\substack{\uparrow \\ \text{\sout{SUSY} terms}}}{+\cdots} \quad (\text{e.g. MSSM, SUSY GUT})$$

$$L_{gauge} = \int d^2\theta \; \tau \; \mathrm{Tr}\,(W_\alpha W^\alpha) + \text{c.c.}$$

$$L_{matter} = \int d^2\theta \, d^2\bar\theta \; \bar\Phi \, e^{gV} \, \Phi + \int d^2\theta \; \underset{\substack{\uparrow \\ \text{holomorphic}}}{W(\Phi)} + \text{c.c.}$$

Chiral superfield

$$\Phi = \underset{\substack{\uparrow \\ \text{cmplx. scalar}}}{\phi} + \theta \underset{\substack{\uparrow \\ \text{spinor partner}}}{\psi} + \theta^2 \underset{\substack{\uparrow \\ \text{Auxiliary.}}}{F}$$

Vector superfield

$$W_\alpha = -\bar D^2 e^V D_\alpha e^{-V} = \tau^a \left(-i\lambda^a_\alpha - i(\sigma^\mu \bar\sigma^\nu \theta)_\alpha F^a_{\mu\nu}\right)$$

$$V = -\theta \sigma^\mu \bar\theta \underset{\substack{\uparrow \\ \text{vector.}}}{A_\mu} + \cdots$$

M5 : worldvolume $\mathbb{R}^3 \times S^{(3)}$

$$\begin{matrix} \mathbb{R}^3 & \times & S^{(3)} \\ \cap & & \cap \\ \mathbb{R}^4 & & X \end{matrix}$$

BPS $\Longleftrightarrow$ S' associative (minimal).

Tension $T = |\Delta W|$

$$\Delta G = G' - G_{flux} = [\hat{S}]$$

$$T = vol(S) \underset{\substack{\uparrow \\ (\text{minimal.}) \\ \leq \mapsto =}}{=} \int_S \Phi = \int_X \Phi \wedge [\hat{S}] = \int_X \Phi \wedge \Delta G$$

$$\Rightarrow \quad W = \int_X \Phi \wedge G + i C_i \wedge G = \int_X (\Phi + iC) \wedge G$$

6/7/05 A. Uranga. New Physics at far infrared. IR modified gravity.

IR Gauge Theory modification.

UV implications of $m_\gamma \neq 0$

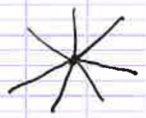
1. GUT in simple grp. $SU(5) \subset SO(10) \subset E_6$

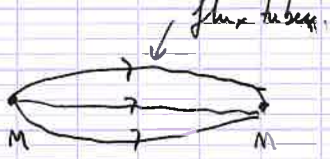
   $T_r Q = 0$ $\langle \phi \rangle \neq 0$ $m_\gamma = g \langle \phi \rangle$ $(g \sim 10^{-10})$

   $V(\phi) = \frac{\lambda^2}{4} (\phi^2 - \eta^2)^2$ $\lambda \simeq 1$.

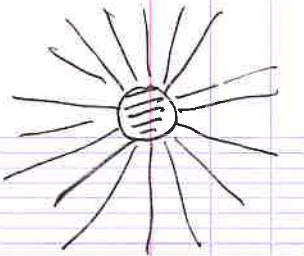
2. Charge quantization?

3. Magnetic monopoles





4/ Black holes



$$A_0 \sim \frac{1}{r} \longmapsto \frac{e^{-mr}}{r}$$ (no Gauss law charge).

PDG '02 limits on photon mass
(from galactic magnetic field)

$m_\gamma \lesssim 10^{-27} eV$ '75 Chibisov

$m_\gamma \leq 10^{-16} eV$ '98 '03 Torque on toroid balance

Gauge invariant theory of photon mass

Proca - Higgs $\partial^\mu F_{\mu\nu} + m_\gamma^2 A_\nu = J_\nu$

Electric Charge Screening

$$\partial^\mu \tilde{F}_{\mu\nu} + m_\gamma^2 \tilde{A}_\nu = J_\nu \rightarrow \partial^\nu \tilde{A}_\nu = 0 \quad 3 \text{ DOF}$$

$$\tilde{A}_\mu = A_\mu - \frac{1}{g} \partial_\mu 4$$

6/8/05 Kachru

Lecture I

Dvali

Lecture II (Lorentz Noninvariant)

$$p^2\left(1+\varepsilon\frac{p_i p_j \cdots p_K}{M_p^N}\right)\psi=0$$ (UV Lorentz violation)

$$\phi\,\Box\,\frac{\partial_{j_1}\cdots\partial_{j_n}}{M_p^N}\,\phi \qquad \frac{S^{j_1}\cdots S^{j_n}}{M_p^N} \qquad \varepsilon\sim\left(\frac{S}{M}\right)^N$$

$\langle S\rangle^4$ ← energy density of universe.

$$\langle S\rangle^4 \lessapprox \rho_c \sim (10^{-3}\mathrm{eV})^4$$

e.g. $\frac{\langle\partial_\mu\chi\rangle}{M}\,\phi\,\Box\,\frac{\partial_\mu}{M}\,\phi$

$\chi(t)$.

$\langle\partial_0\chi\rangle\neq 0$.

$$\langle\partial_0\chi\rangle^2 \lessapprox (10^{-3}\mathrm{eV})^4$$

S could be condensed tensor field.

$$\frac{(10^{-3}\mathrm{eV})^2}{(10^{-17}\mathrm{GeV})^2}$$

$$L=-\frac{1}{4}F^2-m\overset{A_jA^j\ (j=1,2,3)}{A^2}+A\cdot J \qquad \Big\| \qquad \frac{\partial_\mu\phi\,\partial_\nu\phi\,A^\mu A^\nu}{M_p^4}$$ Quintessence Coupling.

$$\partial^\mu F_{\mu\nu}+m^2\delta_\nu^{\ j}A_j=J_\nu \qquad \partial^j A_j=0$$

$\nu=0\rightarrow$ $-\Delta A_0+\cancelto{0}{\partial^j A_0 A_j}=J_0$

$\nu=j\rightarrow$ $(\Box+m^2)A_j-\partial_j\dot{A}_0=J_j$

$$A_j=\frac{1}{\Box+m^2}\left(J_j+\partial_j\dot{A}_0\right)$$

Electric field $E_j=\partial_jA_0-\partial_0A_j=-\frac{\partial_j}{\Delta}J_0-\frac{1}{\Box+m^2}\left(\partial_0 J_j+\cancel{\partial_j\dot{A}_0}\right)$

or $$\mathcal{E}_j = \partial_j A_0 - \frac{1}{\Box + m^2}\left(\partial_0 J_j + \partial_j \dot{A}_0\right)$$

numerator $-\Delta A_0 + m^2 A_0$

$$= -\frac{\partial_0 J_i}{\Box + m^2} - \partial_j\left(\frac{(\Box + m^2)A_0 - \ddot{A}_0}{\Box + m^2}\right) \qquad \text{cf. } A_0 = \frac{J_0}{\Delta}$$

$$= \boxed{-\frac{1}{\Box + m^2}\left(\partial_0 J_i - \partial_j J_0\right) - \underbrace{\frac{m^2}{\Box + m^2}\frac{\partial_j}{\Delta} J_0}}$$

normal term.

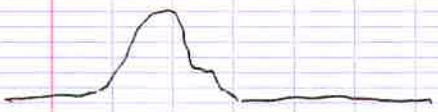
effective source becomes nonlocal. (instantaneous).

this effect is same in every gauge theory: adding small mass gives some seeming nonlocality.

$$(\Box + m^2)\mathcal{E}_j = -\frac{m^2}{\Delta}\partial_j J_0 \qquad \cancel{\Box + m^2} \qquad \dot{\mathcal{E}}_j = \psi_j$$

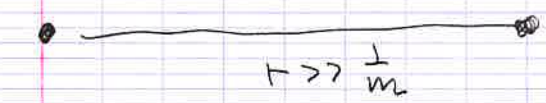
- $J_0 = \delta(x - f(t))\,\delta(y)\,\delta(z)$ $\qquad \psi(t, \vec{p})$

$$\ddot{\psi}_j(t, \vec{p}) + (m^2 + |\vec{p}|^2)\psi_j = -m^2 \underbrace{\frac{P_j P_1}{|\vec{p}|^2} e^{ip_1 f(t)} f(t)}$$



$$\ddot{\psi} + m^2\psi = J(t)$$

$|\vec{p}| \ll m$



(nonlocality).

Exact solution

$J_0 = 4\pi\mu\, \partial_z \delta^3(r)$ dipole at $t=0$.

$E_j = \partial_j \Phi \rightarrow (\Box + m^2)\Phi = -m^2\mu(\partial_z \frac{1}{r})\delta(t)$ $\quad (\frac{1}{\Delta}\delta^3(r) = \frac{1}{r})$

$\Phi = \Phi_\ell + \Phi_a$ $\quad (\ddot{\Phi}_a + m^2\Phi_a = -m\mu^2(\partial_z \frac{1}{r})\delta(t))$

$\uparrow$
local

sol'n $\Phi_a = \mu m(\partial_z \frac{1}{r})\sin(mt)\theta(t)$

$\uparrow$
precursor

$(\Box + m^2)\Phi_\ell = \Delta\Phi$

(immediate response.)

$t=0, r=0$

$\Delta t \sim m^{-1}$

Story is identical in gravity.

c.f. m^2A^2 introduces. $\partial_\mu A^\mu = 0$ $\quad \tilde{A}_\mu = A_\mu - \partial_\mu\psi$

Now: $A_j = (\tilde{A}_j - \partial_j\psi)$

$A_0 = \tilde{A}_0$

$m^2 A_\mu A^\mu \longrightarrow m^2 \partial_\mu\psi\,\partial^\mu\psi$

$m^2 A_j A_j \rightarrow m^2 \partial_j\psi\,\partial_j\psi = m^2[\partial_j\psi\,\partial_j\psi - \frac{1}{c'}\partial_0\psi\,\partial_0\psi]$ w/ $c' \rightarrow \infty$.

Story is identical in gravity.

When local piece gets to precursor it cancels it out.

$m A_j A^j$

$\uparrow$

BOUNDS?

starlight dispersion $\sim 10^{-14} \sim 10^{-16}$

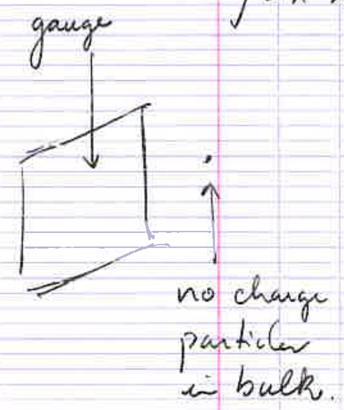
Localization of Particles on a (ridgid) brane.

$$S_{4+N} = \int dx^{4+N} \; \partial_A \chi(x_B) \, \partial^A \chi(x_B)$$

$$+ \int d^4_x \; \partial_\mu \varphi(x_\nu) \, \partial^\mu \varphi(x_\nu)$$

$\uparrow$ Brane Action (Rigid)

$$+ \int d^4_x \; \lambda \varphi^2(x_\nu) \, \chi \; \ldots$$



$\mathcal{C}_2$

String "exception"

$$\int_{3+1} F_2 \wedge C_2 \quad + \quad \int_{4+1} (dC_2)^2$$

$\Big\downarrow$ duality

$$\int_{3+1} -F^2 + (A_\mu - \partial_\mu c)^2 \qquad \partial_\mu a = \varepsilon_{\mu\alpha\beta\gamma} F^{3(\alpha\beta\gamma)}$$

6/8/05

IR gravity

Motivation

1. Cos Const. $h_{\mu\nu}$ $\substack{s=2\\ m=0}$ $\Rightarrow$ GR, couples to anything.

   deviations of GR → change degrees of freedom.

   $$S = \int_{3+1} \sqrt{-g}\,R + (?)$$ r_c (want to change GR at $r \gg r_c$

   graviton may decouple from vacuum energy.

2. Dark Energy.

3. Can you consistently modify GR in IR.

h → additional D.O.F.

$A_\mu \mapsto A_\mu - \partial_\mu \psi$ A∧J cannot emit long. polarization.

$m^2 \to 0$ we recover observations continuously.
but formal theory is continuous.

In GR $m \to 0$ observations are discontinuous.
Fierz Pauli can have mass.

$$m_g\left(h_{\mu\nu}^2 - (h^\alpha_\alpha)^2\right) \leftarrow \text{still useful.}$$

$$\underset{\substack{\uparrow\\ \text{massive}\\ \text{graviton}}}{\partial^\mu h_{\mu\nu} = 0} \qquad \text{~~5 constraints~~} \quad \text{5 D.O.F.}$$

$$h_{\mu\nu} \Rightarrow \overset{\substack{2\\ \downarrow}}{\tilde{h}_{\mu\nu}} \overset{+}{} + \overset{\substack{2\\ \downarrow}}{A_\mu} + \underset{\substack{\uparrow\\ \text{source of trouble.}}}{\overset{\substack{1\\ \downarrow}}{\underline{\underline{\Phi}}}} \qquad 2 + 2 + 1 = 5$$

couples to sources gravitationally. $\boxed{\Phi\, T^\mu{}_\mu}$

1-graviton exchange

Amplitude $= A(p^2) = G_N \dfrac{T_{\mu\nu} T'^{\mu\nu} - \frac{1}{2} T^{\alpha}_{\ \alpha} T'^{\beta}_{\ \beta}}{p^2} \to G_N \dfrac{m_S m_E}{r}$

$$Am(p^2) = G_N \frac{T_{\mu\nu} T'^{\mu\nu} - \left(\frac{1}{2} + \overset{1/6}{\alpha}\right) T^{\alpha}_{\ \alpha} T'^{\beta}_{\ \beta}}{p^2 + \boxed{m_g^2}}$$

$\to 0$ discontinuous

$\partial_\mu A_\nu T^{\mu\nu}$
boundaries ?

$$h_{\mu\nu} T'^{\mu\nu} = \int ds \frac{T_{\mu\nu} T'^{\mu\nu} - \frac{1}{3} T^{\alpha}_{\ \alpha} T'^{\beta}_{\ \beta} \, \rho(s)}{s^2 - p^2} + ?$$

$$G(p) = \int_0^\infty \frac{ds\, \rho(s)}{s - p^2}$$

If theory admits spectral representation — discontinuity exists.

$\rho(s) \geq 0$ positive definite.

$\sim \rho(s)$ pos. def $\Rightarrow$ ghosts

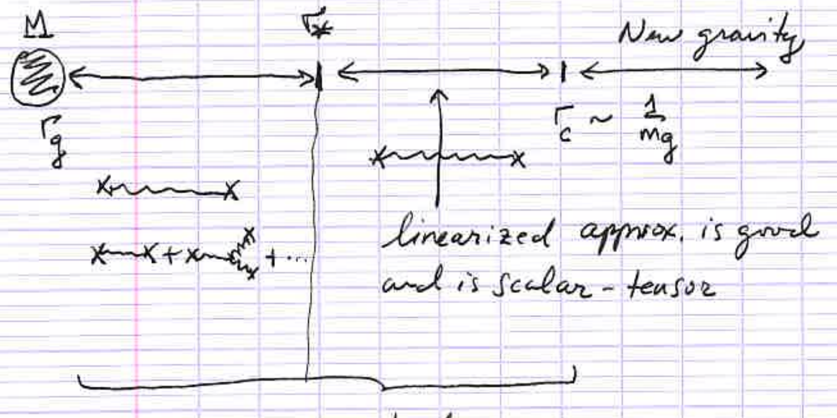
If we modify gravity in IR:

1. $\exists$ Discontinuity (DV? type)
2.

$\boxed{G_N \qquad\qquad\qquad G_N^2}$

<u>Ruled out</u> (If theory is weakly coupled at solar system distances)

Properties of IR gravity: (if it exists)



r_* is source dependent

very robust

$$\boxed{r_g \lesssim r_* \lesssim r_c \qquad r_* \sim (r_g^\alpha r_c^\beta)^{\frac{1}{\alpha+\beta}}}$$

α, β model dependent.

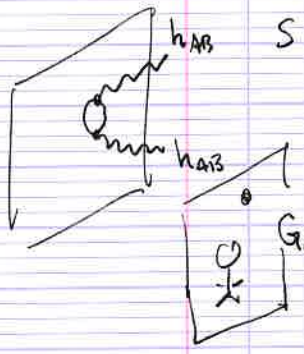
Example:

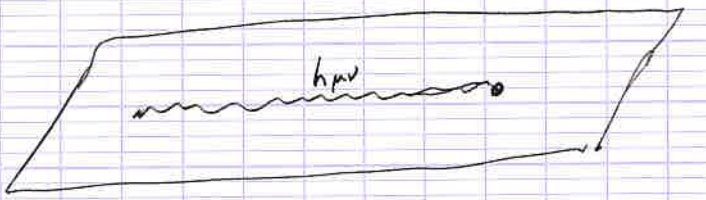
$$S = \int_{4+1} \sqrt{-G}\, \underset{\substack{\uparrow \\ \text{fundamental scale.}}}{M_*^3} R_{(5)} + \int_{3+1} M_p^2 \sqrt{-g}\, R_{(4)}$$

tells $h_{\mu\nu}$ to stay on brane.

$g_{\mu\nu} = \partial_\mu X^A \partial_\nu X^B G_{AB}$

$R_{(4)}(g_{\mu\nu})$





at short distance $h_{\mu\nu}$ stays on brane
at long distance $h_{\mu\nu}$ goes to bulk.

$$M_*^3 \left[\int_{4+1} (\partial_A \phi)^2 + r_c \int_{3+1} (\partial_\mu \phi)^2 \right]$$

$$r_c = \frac{M_p^2}{M_*^3} \qquad \Big(\underbrace{\Box_5}_{\Box_4 - \partial_y^2} + r_c\, \delta(y)\, \Box_4 \Big)\, G(x) = \delta^3(x)\, \delta(y)$$

$G(p, y)$ Fourier.

$$\to \quad (p^2 - \partial_y^2 + r_c \delta(y) p^2) G(p,y) = \delta(y)$$

$$G(p,y) = D(p,y) B(p)$$

where $$(p^2 - \partial_y^2) D(p,y) = \delta(y) \to D(p,y) = \frac{e^{-|y|p}}{2p}$$

$$\left(\delta(y) + r_c \delta(y) p^2 D(p,\underset{\substack{\shortparallel \\ 0}}{y})\right) = \delta(y)$$

$$\left(1 + r_c p^2 D(p,0)\right) B(p) = 1$$

$$B(p^2) = \frac{1}{1 + r_c \frac{p^2}{2p}} \to G(p,y) = \frac{1}{r_c} \frac{e^{-|y|p}}{p^2 + p/r_c}$$

on brane $y = 0 \longrightarrow G(p,0) = \frac{1}{r_c} \frac{1}{p^2 + p/r_c}$

for $p \gg \frac{1}{r_c}$ behaves like $\frac{1}{p^2}$ (4D massless particle)

$p \ll \frac{1}{r_c}$ behaves like $\frac{1}{p}$ (5D massless particle)

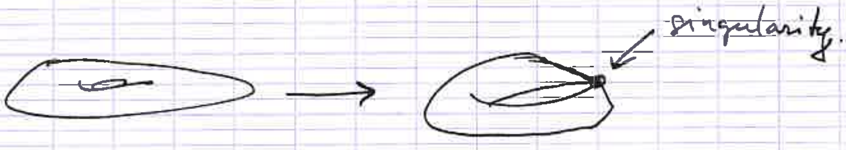
$$\Rightarrow \quad V(r) = \begin{cases} \frac{1}{r} & r \ll r_c \\ \frac{1}{r^2} & r \gg r_c \end{cases}$$

6/8/05 Gukov IV

M / G_2 $\quad b_2$ vectors

b_3 chiral multiplets $\leftarrow \int C + i\Phi$

Singularities

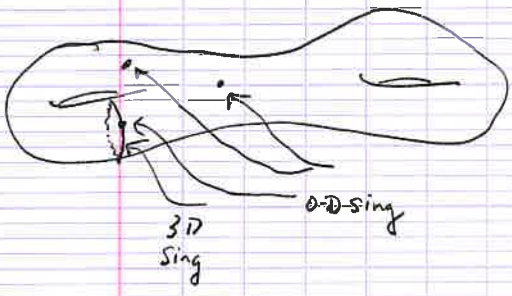


physics that may arise from singularities:

- new degrees of freedom
- extra (nonabelian) gauge groups
- continuous or discrete symmetries
- topology changing transition

⋮

Model building with G_2 manifolds



3D Sing → support gauge fields

0D sing → chiral fermions

BOTH → charged matter.

Strategy:

- use various dualities
- branes wrapped on supersymmetric cycles.
- resolve singularities

Nonabelian gauge fields:

M-theory / Het. String duality

M-theory on K3 $\equiv$ Het on 3D torus (T^3).

- same supersymmetry
- moduli match
- spectrum of BPS states ...

in Het. String $\oint_\gamma A$ ⟵ turning on Wilson lines
$\rightsquigarrow$ 7D gauge ~~group~~.

$SO(32)$; $E8 \times E8$

→ K3 must be singular

$$X \cong \mathbb{C}^2 / \Gamma_{ADE} \qquad \Gamma_{ADE} \subset SU(2) \longleftrightarrow \text{ADE type gauge symmetry}$$

$$A_{N-1} : SU(N) \to \mathbb{C}^2/\mathbb{Z}_N \quad \mathbb{Z}_N : \begin{pmatrix} z^1 \\ z^2 \end{pmatrix} \to \begin{pmatrix} e^{\frac{2\pi i}{N}} z_1 \\ e^{-\frac{2\pi i}{N}} z_1 \end{pmatrix}$$

$$\underline{X} \qquad x^N + y^2 + z^2 = 0 \quad , \{x, y, z\} \in \mathbb{C}^3$$

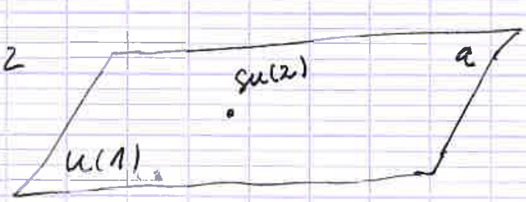
<u>Exc.</u> $\{x, y, z\} \in \mathbb{R}^3$ draw $\underline{X}$.

<u>resolution</u>: $\tilde{X}$ ⟵ the resolution

$$\tilde{X} : \prod_{i=1}^{N} (x - a_i) + y^2 + z^2 = 0$$

↑ moduli

<u>Ex</u>. $N = 2$



"Compactify" 7D thy → 4 dims. on $W^{(3)}$. Choose $W^{(3)} = S^3$

to get $N=1$ SUSY ⇒ we need nontrivial fibration of K3 over S^3:

$$\begin{tikzcd} \bar{X}_{G_2} \arrow[d] & \left\{ \begin{matrix} K3 \\ \downarrow \\ S^3 \end{matrix} \right. & & \left. \begin{matrix} T^3 \\ \downarrow \\ S^3 \end{matrix} \right\} & CY_3 \arrow[d] \\ \mathrm{Hol} = G_2 & & & & \mathrm{Hol} = SU(3) \end{tikzcd}$$

$$\underline{X} \left\{ \begin{matrix} \mathbb{C}^2/\mathbb{Z}_N \\ \downarrow \\ S^3 \end{matrix} \right. \qquad \boxed{\underline{X} \cong \mathbb{C}^2/\mathbb{Z}_N \times S^3}$$ → gives rise to $su(N)$ gauge theory.

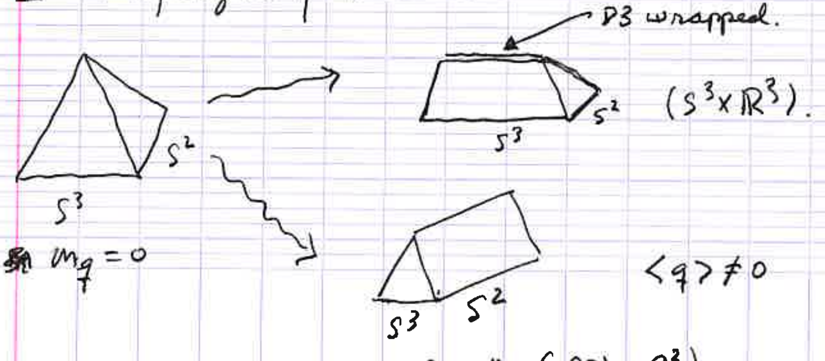
coupling: $$\tau = \frac{\theta}{2\pi} + i\frac{4\pi}{g^2} = \int_{S^3} (C + i \underset{\substack{\uparrow \\ \mathrm{Vol}(S^3)}}{\underline{\phi}})$$

Conclusion: singularity is worse in limit $\mathrm{Vol}(S^3) \to 0$ (strong coupling limit)

Calabi-Yau singularities:

- $\dim_{\mathbb{C}} X = 2$ ADE $\mathbb{C}^2/\Gamma_{ADE}$ ✓
- $\dim X = 3$ $\underline{X}$ = cone on $\underline{Y}$; $\dim \underline{Y} = 5$

$\underline{Y}$ is topologically $S^2 \times S^3 \cong T^{3,1}$



$m_q = 0$

$(S^3 \times \mathbb{R}^3)$.

$m_q \neq 0 \longrightarrow 0$ at conifold point.

2 desingularizations.

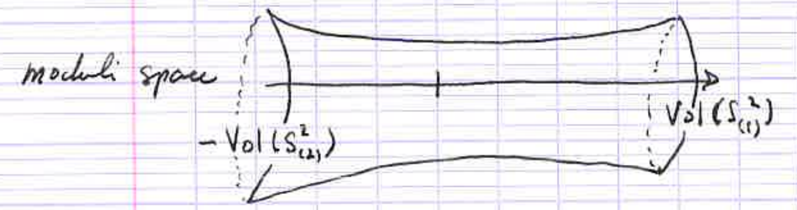
$\langle q \rangle \neq 0$

$S^2 \times \mathbb{R}^4$ $(\mathbb{CP}^1 \times \mathbb{C}^2)$

Topology changing transitions:

* conifold transition: $S^3 \longrightarrow \cdot \longrightarrow S^2$ phase transition

* flop: $S^2_{(1)} \longrightarrow \cdot \longrightarrow S^2_{(2)}$ smooth

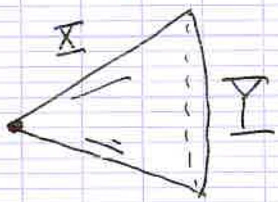


Witten, Greene, Aspinwall

string instantons desingularize geometry

Lets now study G_2 singularities:

* Conical singularities



[singularities → largest co-dimension possible.]

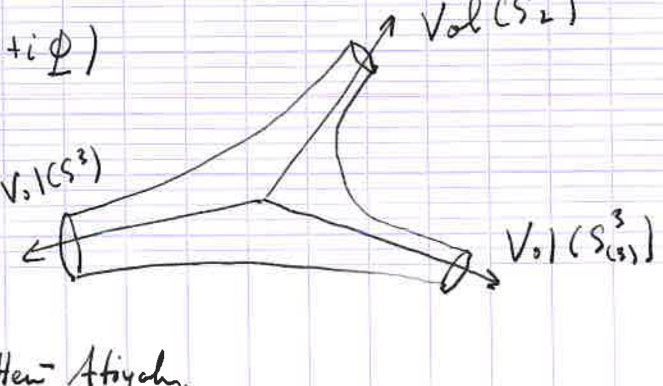
$$Y = S^3 \times S^3 ,\ \mathbb{CP}^3 ,\ SU(3)/U(1)^2$$

* $Y = S^3 \times S^3$

analogy of flop.

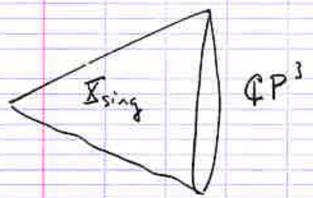
$$\underline{X} \simeq S^3 \times \mathbb{R}^4$$

$$\cancel{\Phi = \int_{S^3}} \qquad Q = \int_{S^3} (C + i\Phi)$$



|| Witten-Atiyah.
AMV

$* \; Y = \mathbb{C}P^3$

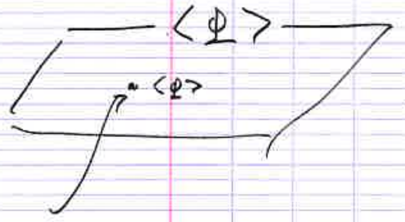


$\rightsquigarrow \quad X \cong S^4 \times \mathbb{R}^3$

↳ chiral superfield

$$\int_{\mathbb{R}^3} (C + i\Phi) = \Phi$$

not the same.

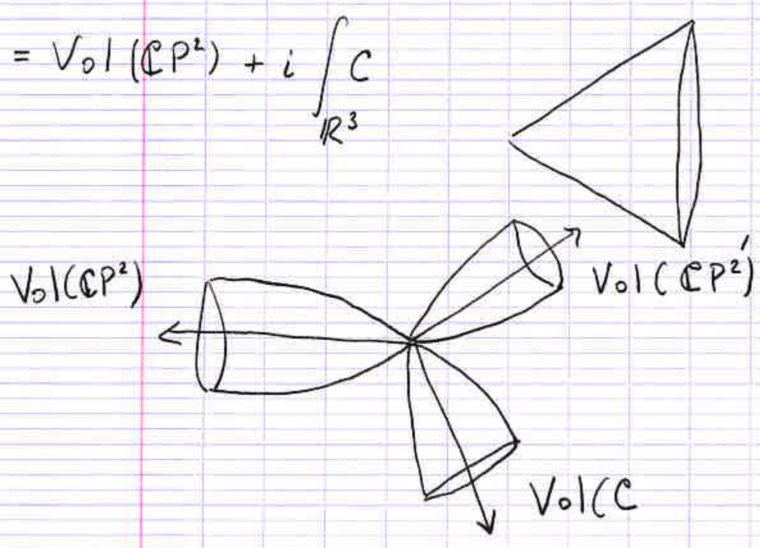


extra U(1) global symmetry.

$\Phi \to e^{i\alpha}\Phi$

$* \; Y \cong SU(3)/U(1)^2 \qquad X \cong \mathbb{C}P^2 \times \mathbb{R}^3$

$$\Phi = \mathrm{Vol}(\mathbb{C}P^2) + i \int_{\mathbb{R}^3} C$$



- 3 branches
- triality symmetry
- phase transition

Kachru -

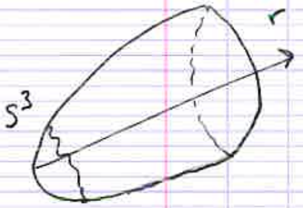
See Silverstein TASI '99 for scaling argument.

Gukov 6/9/05

Dynamics of strongly coupled gauge theories. w/4 supercharges

* $\mathbb{X} = S^3 \times \mathbb{R}^4$ BS '89 GPP '89

$$ds^2(\mathbb{X}) = \frac{dr^2}{1-\left(\frac{r_0}{r}\right)^2} + \frac{r^2}{12}\sum_{a=1}^{3}(\sigma_a - \Sigma_a)^2 + \frac{r^2}{36}\left(1-\frac{r_0^2}{r^2}\right)\sum_{a=1}^{3}(\sigma_a+\Sigma_a)^2$$



$$d\sigma_a = -\frac{1}{2}\varepsilon_{abc}\,\sigma_b\wedge\sigma_c \qquad d\Sigma_a = -\frac{1}{2}\varepsilon_{abc}\,\Sigma_b\wedge\Sigma_c$$

* $\mathbb{X} = S^4 \times \mathbb{R}^3$, $\mathbb{C}P^2 \times \mathbb{R}^3$

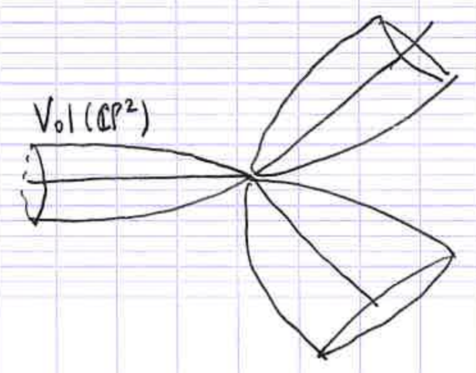
$$ds^2(\mathbb{X}) = \frac{dr^2}{\left(1-\frac{r_0^2}{r^2}\right)} + \frac{r^2}{4}\left(1-\left(\frac{r}{r_0}\right)^4\right)|d_A u|^2 + \frac{r^2}{2}ds^2(M)$$

$$d_A u_i = du_i + \varepsilon_{ijk}A_j u_k$$

$$\sum_{i=1}^{3} u_i^2 = 1$$

$$ds^2(X) = dr^2 + r^2 ds^2(\mathbb{X})$$

* $\mathbb{X} = \mathbb{C}P^2 \times \mathbb{R}^3$



type II string theory on $\underline{X}$ $\phi = \mathrm{vol}(\mathbb{CP}^2)$

$C = \sigma\, \omega^{(3)}$

15

vacuum structure depends weakly on dimensionality $4 \sim 3$.

$\mathrm{IIA}/\underline{X} \longrightarrow \mathcal{N}=2$ 3D QFT

$\phi, \sigma \quad d\sigma = *_3 dA \Rightarrow *_3 d\sigma = dA$

(A, ϕ) $\mathcal{N}=2$ vector multiplet

Chiral superfield:

1 complex scalar + fermions + ...

Vector superfield

$A_\mu \Rightarrow (A, \phi)$

Light states due to D-branes wrapping cycles:

Effective $\mathcal{N}=2$ theory: $U(1)$ vector multiplet (A, ϕ) and two charged matter multiplets. $q_+, \tilde{q}_-$

$q_+, \tilde{q}_-$ come from D4 brane on $\mathbb{CP}^2 \subset \underline{X}$. As ~~$\mathbb{CP}^2 \to 0$~~ $\mathrm{vol}(\mathbb{CP}^2) \to 0$ we get light DOF's. $U(1)$ gauge field on D4 brane $\int_{\mathbb{CP}^1} \frac{F}{2\pi} \in \mathbb{Z}_1$

$\mathbb{CP}^2$ is not spin $\Rightarrow \left[\frac{F}{2\pi}\right] - \frac{W_2}{2} \in H^2(\mathbb{CP}^2; \mathbb{Z})$ Freed-Witten anomaly.

$H_k(\mathbb{CP}^2) = \begin{cases} 1 & k = 0,2,4 \\ 0 & \text{other} \end{cases}$

~~Field strength~~

$$\Rightarrow \int \frac{F}{2\pi} \in \mathbb{Z}_1 + 1/2$$

$$\int_{\mathbb{CP}^1} \frac{F}{2\pi} = k \in \mathbb{Z}_1 + 1/2 \qquad M_{D4} = \mathrm{vol}(\mathbb{CP}^2) - \frac{1}{2}\int_{\mathbb{CP}^2} \frac{F}{2\pi} \wedge \frac{F}{2\pi}$$

$$+ \frac{1}{48}\int_{\mathbb{CP}^2} \left(p_1(T\mathbb{CP}^2) - p_1(N\mathbb{CP}^2) \right)$$

c.f. WZW

$$I_{WZ} = \int C_x \wedge ch(F) \wedge \sqrt{\frac{\hat{A}(TM)}{\hat{A}(NM)}}$$

$$\hat{A} = 1 - \frac{p_1}{24} + \frac{7p_1^2 - 4p_2}{5760}$$

$$\rightsquigarrow M_{D4} = \Phi + \frac{1}{2}K^2 - \frac{1}{8} \geq \Phi \qquad (= \text{ for } K = \pm\tfrac{1}{2})$$

$\rightarrow$ $K = \pm\frac{1}{2}$ correspond to $q_+, \tilde{q}_-$

3D $\mathcal{N}=2$ SQED w/ $N_f = 2$ flavours:

U(1) $(A, \phi) \longleftrightarrow (\sigma, \phi)$ superpotential $V = e^2(|q|^2 - |\tilde{q}|^2) + \phi^2(|q|^2 - |\tilde{q}|^2)$

$q \rightarrow$ chg $+1$

$q \rightarrow$ chg -1

D-term

in 4D $\int d^4\theta\, Q^\dagger e^V Q$

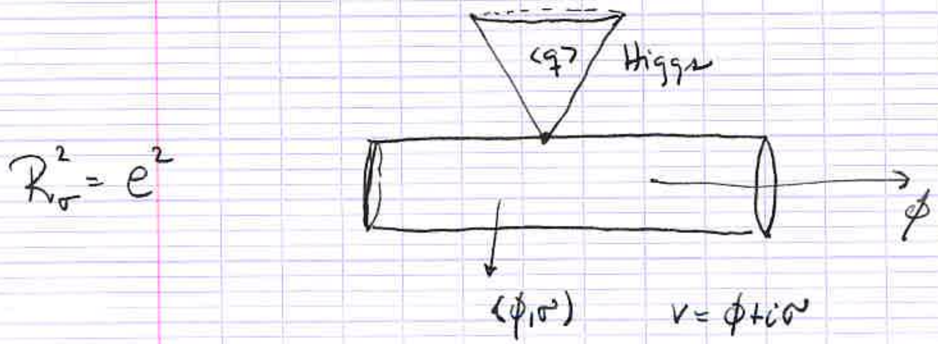
$V = \cancel{A_\phi}\, \phi\theta\theta + \cdots$

Classical moduli space:

$\underline{V = 0}$:

Coulomb branch: $\langle q \rangle = \langle \tilde{q} \rangle = 0$ $\quad \langle \phi \rangle \neq 0$

$\langle \sigma \rangle \neq 0$



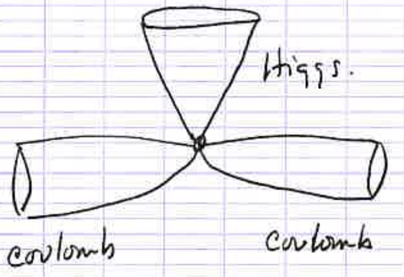
Higgs branch: $\langle \phi \rangle = 0$ $\langle q \rangle \neq 0$ $\langle \tilde{q} \rangle = 0$ divide by $U(1)$

→ 1-complex dimensional

σ is pure gauge on Higgs branch.

$$R_\sigma^2 = e^2 \underset{\text{quantum effects}}{\rightsquigarrow} \left(\frac{1}{e^2} + \frac{1}{\phi}\right)^{-1}$$

(for ~~N~~ $\overbrace{8 \text{ supercharges}}^{N=4}$ this would be the total).



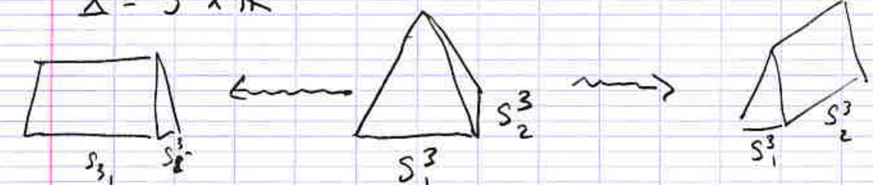
At center → Landau Ginsburg theory.

$W = M V_+ V_-$

//

Ex 2 4D strongly coupled theory.

$X = S^3 \times \mathbb{R}^4$



$X = S^3_{(1)} \times \mathbb{R}^4$

$X = S^3_{(2)} \times \mathbb{R}^4/\mathbb{Z}_N$

↓

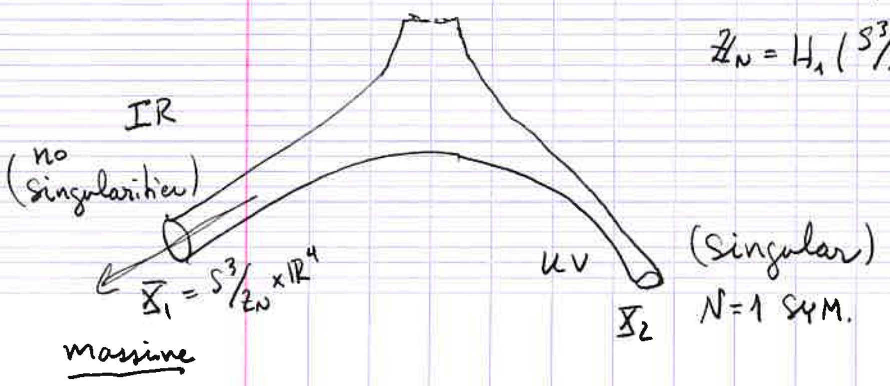
$S^3_{(1)}/\mathbb{Z}_N$ by acting $\mathbb{C}^2 \simeq \mathbb{R}^4$ has S^3_∞

$\mathbb{Z}_N : \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \to \begin{pmatrix} e^{2\pi i/N} z_1 \\ e^{-2\pi i/N} z_2 \end{pmatrix}$

Lenz space smooth.

confining strings

$\mathbb{Z}_N = H_1(S^3/\mathbb{Z}_N)$



⟹ $N=1$ SYM has a mass gap.

Exc. (Intersecting Brane Models)

Het - M-thy duality to get nonabelian gauge fields

$\mathbb{II}A \simeq$ M-thy$/S^1$

RR 1-form

M-thy on G_2 manifold $\underline{X}$ $\longleftrightarrow$ $\mathbb{II}A$ on $\underline{X}/U(1)$ w/ D6 branes 2 form RR fields

$M^6 = X^7/U(1) \leftarrow$ CY

$\boxed{\text{M thy}\big|_{G_2}} \simeq \boxed{\mathbb{II}A \text{ on } X/U(1) \text{ w D6-branes on } L \subset M^6}$

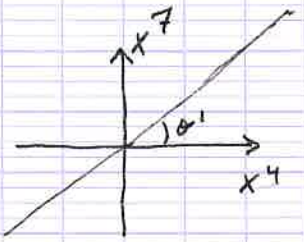
$\uparrow$

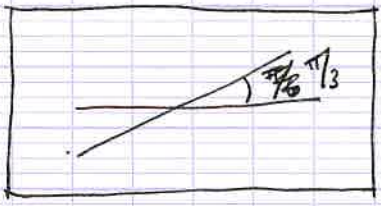
SLAG

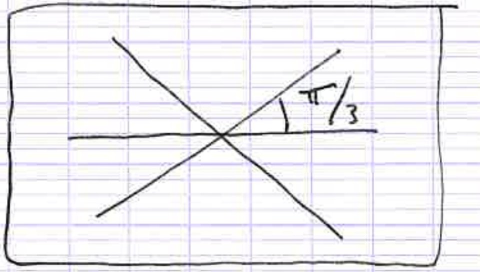
$M^6 \simeq X/U(1) \simeq \mathbb{R}^6 \simeq \mathbb{C}^3$ D6 branes on $L \subset \mathbb{R}^6$

Consider:

$$D6 : 0123 \underset{\theta_1}{\begin{bmatrix}4\\7\end{bmatrix}} \underset{\theta_2}{\begin{bmatrix}5\\8\end{bmatrix}} \underset{\theta_3}{\begin{bmatrix}6\\9\end{bmatrix}}$$







which one goes with which?

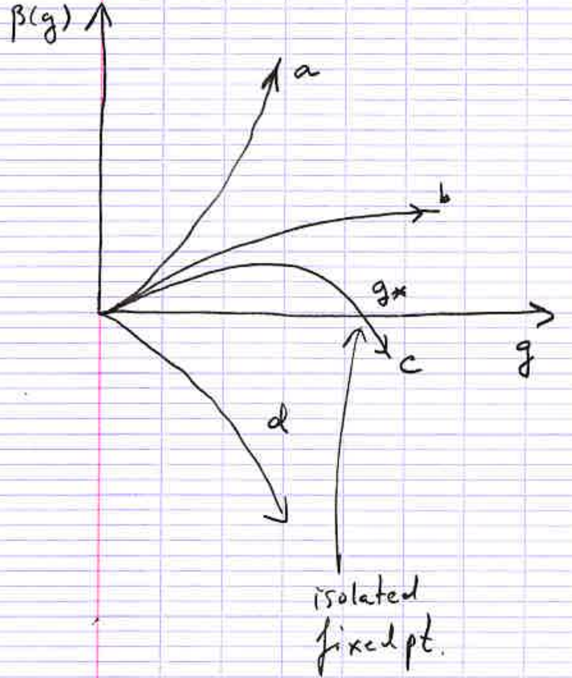
6/13/05

Klebanov I

Cascading gauge theory and string theory duals

I. Klebanov.

~~Suggested reading~~ –



a) Sing. @ finite E

$$\int_{g_\mu}^{\infty} \frac{dg}{\beta(g)} < \infty \qquad E_\infty = \mu \int_{g_\mu}^{\infty} \frac{dg}{\beta(g)}$$

b) Continued growth

$$\int \frac{dg}{\beta} \text{ blows up.}$$

c) Fixed UV stable pt.

finite g_* : $\beta(g) \to a(g_* - g)$

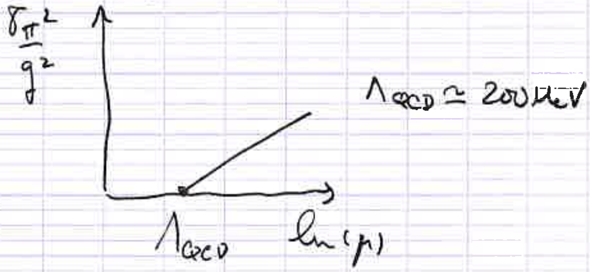
$$g_\mu = g_* - \text{const}\, \mu^{-a}$$

d) Asymptotic freedom in QCD coupled to n_f light flavours

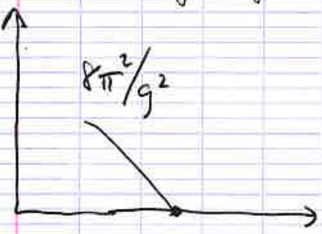
$$\mu \frac{\partial g}{\partial \mu} = \frac{-g^3}{(4\pi)^2}\left(\frac{11}{3} C_2(G) - \frac{2}{3} n_f\right);$$

$\overset{\shortparallel}{T_2(\text{adj})}$ (= N for SU(N))

$$\mu \frac{\partial}{\partial \mu} \frac{8\pi^2}{g^2} = \frac{4}{3} N - \frac{2}{3} n_f^2$$



If $n_f > \frac{11}{2} N$



fixed pt → fixed line.

Conformal fixed lines:

β-fn. cancels order by order in perturbation theory.

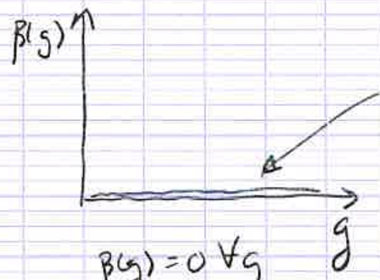
N=4 SYM theory:

field content: 4 adj Weyl fermions
6 adj scalars

1-loop β-coefficient: $b_1 = \frac{11}{3} C_2(G)$

$$- \frac{2}{3} \sum_F T_2(F) - \frac{1}{6} \sum_S T_2(S)$$

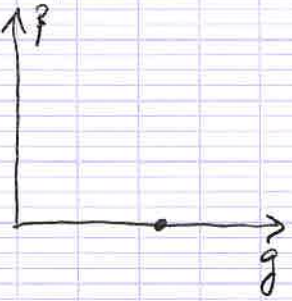
$$= T_2(adj) \left(\frac{11}{3} - \frac{8}{3} - \frac{1}{6} \cdot 6 \right) = 0$$



theory does not flow.

$\beta(g) = 0 \ \forall g$

Conformal unification



E → ∞ ⟶ fixed line

Cascade: Closer & closer to conformality w/o quite getting there

(RG limit cycle ~~of F~~)

more like a

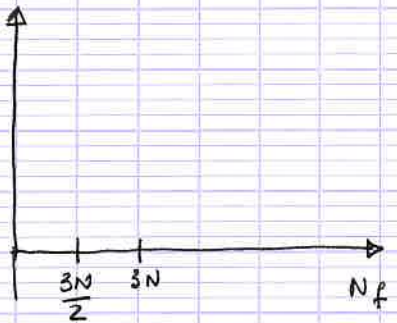
"limit spiral"

Seiberg duality: (see Strassler)

IR equivalence of pairs of $N=1$ gauge theories.

A: SQCD $N=1$ gauge theory with $SU(N)$ coupled to fields Q^r in N of $SU(N)$, $r=1,\dots,N_f$ $\tilde{Q}_u$ in $\bar{N}$ of $SU(N)$ $u=1,\dots,N_f$.

$SU(N_f)_L \times SU(N_f)_R$ flavour symmetry.



Non-anomalous $U(1)_R$; gluinos λ have R charge 1.

~~Qs~~ → R charge $1-\frac{N}{N_f}$ for squarks

→ fermions $-\frac{N}{N_f}$ = R charge.

$\frac{3N}{2} < N_f < 3N$ conformal window.

$\tilde{Q}$'s ↓

$U(1)_R$ anomaly $\sum'_{\text{ferm}} T_2(F) R = N - \frac{1}{2} N_f \frac{N}{N_f} - \frac{1}{2} N_f \frac{N}{N_f} = 0$ ✓

B: (magnetic dual) SQCD + M(eson)

from A = SQCD

$G = SU(\tilde{N})$ $\tilde{N} = N_f - N_c$

N_f flavours q_r of R-charge $1-\frac{\tilde{N}}{N_f}$

$\tilde{q}_u$

Gauge singlet meson

↓

$SU(\tilde{N})$ $\boxed{M^r_u}$ of R charge $\frac{2\tilde{N}}{N_f}$

Superpotential: $W \sim M^r_u \, q_r \, \tilde{q}^u$

$\int d^2\theta \, W$ Marginal $W \Rightarrow$ R charge 2.

$$2\left(1-\frac{\tilde{N}}{N_f}\right) + \frac{2\tilde{N}}{N_f} = 2.$$

Matching of A & B: $M^r_u = Q^r \tilde{Q}_u$

R charge: $\quad 2\left(1-\frac{N}{N_f}\right) \qquad 2\left(1-\frac{N}{N_f}\right)$

In certain theories dimensions of operators are determined by R-charge

$\dim \mathcal{O} = \frac{3}{2} R_{\mathcal{O}}$ ~~dim O~~, in d-spacetime dimensions, unitarity bound $\Rightarrow$

$$\dim \mathcal{O} \geq \frac{d-2}{2}$$

Theory A:

$\dim Q\tilde{Q} = 3\left(1-\frac{N}{N_f}\right) \geq 1 \Rightarrow \boxed{N_f \geq \frac{3N}{2}}$ (lower conformal window boundary)

In magnetic theory (B) higher bound.

$\dim (q\tilde{q}) \geq 1 \quad N_f \geq \frac{3\tilde{N}}{2} = \frac{3}{2}(N_f - N) \Rightarrow \boxed{N_f \leq 3N}$

SQCD (large flavours) $\longleftrightarrow$ SQCD + M

≀

colors

How the cascade works:

Question: (BF window in gravity ⟷ ~~cont~~ Conformal window)

$-4 \leq (mL)^2 \leq -3$

$\underbrace{1 \leq}\ \Delta \leq 3$

↑ unitarity bound

$\mathcal{N}=1$ gauge theory

Gauge group $SU(N) \times SU(N)$ $\qquad W = \varepsilon^{ij}\varepsilon^{kl}\,\mathrm{Tr}(A_i B_k A_j B_l)$

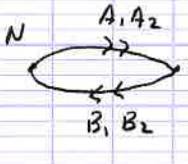
$A_1\ A_2$	N	$\bar{N}$
$B_1\ B_2$	$\bar{N}$	N

- unique $SU(2) \times SU(2)$ invariant expression.

rotates $\begin{pmatrix} A_1 \\ A_2 \end{pmatrix}$ $\qquad$ rotates $\begin{pmatrix} B_1 \\ B_2 \end{pmatrix}$

A bifundamental $A^{\overset{SU(N)_1}{\alpha}}{}_{\underset{SU(N)_2}{\beta}}$

Quiver diagram: nodes - gauge grps.
lines - bifundamental



$A_1 \sim N\ Q$'s
$B_1 \sim N\ \tilde{Q}$'s
→ Each gauge group effectively has $N_f = 2N$ flavours.
R charge of A's, B's is $\frac{1}{2}$

$R_W = 2 \Rightarrow$ Superconf. as explained.

This theory is self-dual under Seiberg duality.

break conf. inv. $SU(N)\times SU(N) \longmapsto SU(N+M)\times SU(N)$

$(M \ll N \Rightarrow$ small breaking of conf. inv.$)$

Shifman - V β - fn. SQCD

gluino

$\frac{11}{3}N - \frac{2}{3}N$

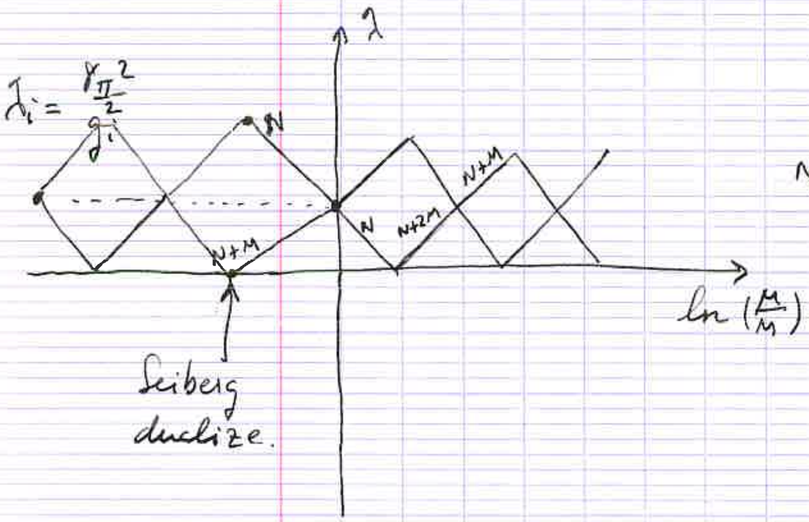
$$\mu \frac{d}{d\mu} \frac{8\pi^2}{g^2} = 3N - N_f(1-\gamma) \equiv \text{anomaly coefficient of } U(1)_R$$

anomalous dim of A_iB_j

$$\gamma = -\frac{1}{2} + a\left(\frac{M}{N}\right)^2$$

$$\Rightarrow \mu \frac{d}{d\mu} \frac{8\pi^2}{g_1^2} = 3(N+M) - 2N(1+\tfrac{1}{2}) = 3M \quad \text{— Asy-p-free}$$

$$\mu \frac{d}{d\mu} \frac{8\pi^2}{g_2^2} = 3N - 2(N+M) = -3M \quad \text{— IR free}$$



$\lambda_1 + \lambda_2 = \text{const.}$

$N_{eff} \sim \ln\left(\frac{E}{\Lambda_{QCD}}\right)$

Seiberg dualize at singular point.

$$\tilde{N} = N_f - (N+M) = 2N - N - M = N - M$$

after duality $\longrightarrow SU(\tilde{N}-M)\times SU(\tilde{N})$

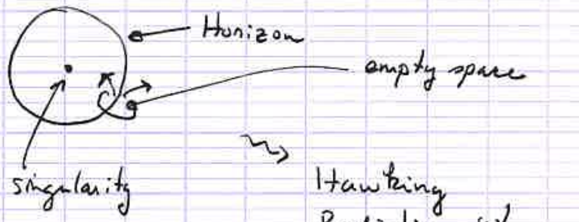
$\tilde{N} = N - M$

Black Holes in string theory

Samir Mathur

Information paradox

Dust Cloud



Radiation does not have information about the matter.

$$i\frac{\partial\psi}{\partial t} = H\psi \qquad \left.\begin{aligned}\psi_f &= e^{-iHt}\psi_i \\ \psi_i &= e^{iHt}\psi_f\end{aligned}\right\}$$

⇒ Violation of quantum mechanics.

Some assumption must be wrong.

i) Classical gravity + classical matter → No radiation
ii) " + quantum matter → Radiation + paradox.
iii) quantum grav + quantum matter → should resolve.

usually Quant grav. $l \sim l_p$.

Hawking theorem:
- All QG effects are confined to $l \lesssim l_p, l_s \cdots$
- Vacuum is unique.

⇒ Information loss

Black Holes in string theory: . Entropy of black holes

. Structure of black holes

. AdS/CFT "hair" for bh's.

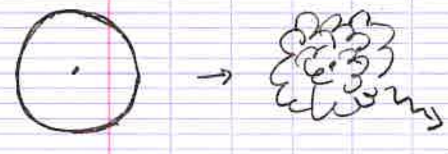
Seems AI is wrong

$G, \hbar, c$

$\underline{\underline{l_p}} = \sqrt{\frac{G\hbar}{c^3}} \sim 10^{-33}$ cm.

but.. | Black hole ~ large # of quanta
quantum gravity length scale might grow
with N.?

$$\boxed{l \sim l_p N^{\alpha}}$$



radiation ~~does~~ does
have info about matter.

Review Hawking arguement:

$G = \hbar = c = 1$

Schwarzschild metric in 3+1

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega^2 \quad f(r) = 1 - \frac{2M}{r}$$

$r = 2M$ coordinate singularity

Good coords

r_*:

$$-(1-\tfrac{2M}{r})\left[-dt^2 + dr^2\,(1-\tfrac{2M}{r})^2\right] + r^2 d\Omega^2$$

$$\frac{dr}{(1-\frac{2M}{r})} = dr^*$$

$$\Rightarrow \quad r^* = \int dr\,(1-\tfrac{2M}{r})^{-1} = \int dr\,\frac{r}{r-2M} = r + \int dr\,\frac{2M}{r-2M}$$

$$= r + 2M\ln(r-2M) + C$$

$$= r + 2M\ln(\tfrac{r}{2M} - 1)$$

$$ds^2 = \left(1 - \frac{2M}{r}\right)\left[-dt^2 + dr^{*2}\right] + r^2 d\Omega^2$$

$$u = t + r^* \qquad du = dt + dr^*$$

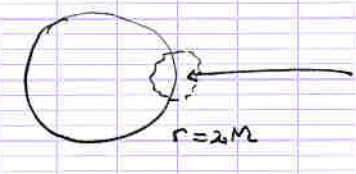
$$v = t - r^* \qquad dv = dt - dr^*$$

$$ds^2 = -\left(1 - \frac{2M}{r}\right) du\, dv + r^2 d\Omega$$

$$r = 2M + \varepsilon \qquad r^* = (2M + \varepsilon) + 2M \ln\left(1 + \frac{\varepsilon}{2M} - 1\right) \sim 2M \ln\left(\frac{\varepsilon}{2M}\right)$$

$$\varepsilon \to 0 \qquad u \to -\infty$$

$$r^* \to -\infty \qquad v \to \infty$$



$$U = e^{u/L} \qquad \boxed{\begin{aligned} dU &= \frac{1}{L} U\, du \\ dV &= -\frac{1}{L} V\, dv \end{aligned}}$$

$$V = -e^{-v/L}$$

$$du\,dv = dU\,dV = L^2 e^{(v-u)/L}$$

$$\boxed{du\,dv = L^2 e^{-2r^*/L}\, dU\, dV}$$

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dU\, dV\, L^2 e^{-2\frac{r^*}{L}} + r^2 d\Omega^2$$

$$\frac{r-2M}{r}$$

$$\underbrace{\frac{2M}{r}}\left(\frac{r}{2M} - 1\right)$$

$$e^{-\frac{2}{L}\left(r + 2M \ln\left(\frac{r}{2M} - 1\right)\right)} = e^{-\frac{2r}{L}} e^{-\frac{4M}{L}\ln\left(\frac{r}{2M} - 1\right)}$$

$$= \underbrace{e^{-\frac{2r}{L}}}\left(\frac{r}{2M} - 1\right)^{-\frac{4M}{L}}$$

Need $\frac{4M}{L} = 1 \Rightarrow \boxed{L = 4M}$

Kruskal $\Rightarrow$ $U = e^{u/4M}$ $\Rightarrow ds^2 = -\frac{2M}{r}(16M^2) e^{-2r/L} dU\, dV + r^2 d\Omega^2$

$V = -e^{-v/4M}$

$$\boxed{ds^2 = -\frac{32M^3}{r} e^{-r/2M}\, dU\, dV + r^2 d\Omega^2}$$

$t, r, \theta, \varphi \to u, v, \theta, \phi$

$$-UV = e^{\frac{u-v}{4M}} \qquad u = t + r^*$$

$$= e^{\frac{2r^*}{4M}} \qquad v = t - r^*$$

$$= e^{r/2M}\left(\frac{r}{2M} - 1\right)$$

$r = 2M$ $\left.\begin{array}{l} u = 0 \\ v = 0 \end{array}\right\}$ Horizon

$$-uv = e^{r/2M}\left(\frac{r}{2M} - 1\right)$$

Singularity $r = 0$ $uv = 1$.

Penrose diagram: skip

Particle creation:

Quantum fields in curved space

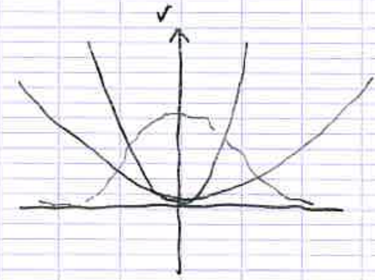
Harmonic oscillator

$$V = \frac{1}{2}kx^2$$

$$|n\rangle = \psi_n(x) = \left[\left(\frac{m\omega}{\pi}\right)^{\frac{1}{4}} \frac{1}{\sqrt{2^n n!}}\right] H_n(\sqrt{m\omega}\, x) \times e^{-m\omega/2\, x^2}$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$E = \hbar\omega + \frac{1}{2}$$



$$|0\rangle_\omega = \left(\frac{m\omega}{\pi}\right)^{\frac{1}{4}} e^{-\frac{m\omega}{2}x^2}$$

$t = 0$ $V = \frac{1}{2}kx^2 \to \frac{1}{2}k'x^2$

$\omega \to \omega'$

$E \to E'$

$|n\rangle_{\omega'}$ are also complete.

during this sudden transition $k \to k'$ $|0\rangle_\omega$ remains same between $t = 0^+$, $t = 0^-$

$$\delta\psi = -iH\psi\delta t = 0$$

$$|0\rangle_\omega = \sum_n{}' C_n |n\rangle_{\omega'}$$

$\uparrow$

State which you have is not the vacuum of new oscillator $|0\rangle_{\omega'}$.

$$\underset{\omega}{[a, a^+] = 1} \qquad \underset{\omega'}{[b, b^+] = 1}$$

$$\hat{a}|0\rangle_\omega = 0$$

$$\left.\begin{aligned} \hat{x} &= \frac{1}{\sqrt{2m\omega}}(a + a^+) \\ \hat{p} &= -i\sqrt{\frac{m\omega}{2}}(a - a^+) \end{aligned}\right| \begin{aligned} \hat{x} &= \left(\begin{matrix}\omega \to \omega' \\ a \to b\end{matrix}\right) \\ \hat{p} &= \left(\begin{matrix}\omega \to \omega' \\ a \to b\end{matrix}\right) \end{aligned}$$

$$\hat{x}\psi(x) = x\psi(x)$$
$$\hat{p}\psi(x) = -i\partial_x\psi(x)$$

Solve for $\hat{a}$ in terms of $\hat{b}$: $\hat{a} = \frac{1}{2}\left(\sqrt{\frac{\omega}{\omega'}} + \sqrt{\frac{\omega'}{\omega}}\right)\hat{b}$

$$+ \frac{1}{2}\left(\sqrt{\frac{\omega}{\omega'}} - \sqrt{\frac{\omega'}{\omega}}\right)\hat{b}^+$$

$\Rightarrow$ (since $a|0\rangle_\omega = 0$) $\Rightarrow \left[\sqrt{\frac{\omega}{\omega'}}(\hat{b} + \hat{b}^+) + \sqrt{\frac{\omega'}{\omega}}(\hat{b} - \hat{b}^+)\right]|0\rangle_\omega = 0$

$c\, e^{\mu b^+ b^+}|0\rangle_{\omega'} = |0\rangle_\omega$ ← guess

$$b\, e^{\mu b^+ b^+}|0\rangle_{\omega'} = 2\mu b^+ \{1 + \mu b^+ b^+ + \frac{\mu^2}{2!}(b^+ b^+)^2 + \cdots]|0\rangle_{\omega'}$$
$$= 2\mu b^+ e^{\mu(b b^+)}|0\rangle_{\omega'}$$

$$\sqrt{\frac{\omega}{\omega'}}\cdots \quad \sqrt{\cdots}\cdots$$

$$\uparrow \quad \nearrow$$

$$(Ab+Bb^+)\,e^{\mu bb^+}|0\rangle_{\omega'}=0$$

$$A\,2\mu b^+ e^{\mu b^+b^+}|0\rangle_{\omega'}$$

$$+B\hat{b}^+ e^{\mu b^+b^+}|0\rangle_{\omega'}=0$$

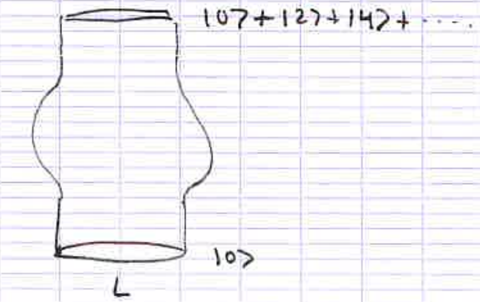
$$\mu=-\frac{B}{2A}$$

$$\mu=-\frac{1}{2}\frac{(\omega-\omega')}{\omega+\omega'}$$

$$\hat{\phi}=\sum_k{}' \frac{1}{\sqrt{V}}\frac{1}{\sqrt{2\omega_k}}a_k e^{i(kx-\omega t)}+\frac{1}{\sqrt{V}}\frac{1}{\sqrt{2\omega_k}}a_k^+ e^{-i(kx-\omega t)}$$

$$[a_k,a_k^+]=1$$

$$\omega_k=\sqrt{k^2+m^2}$$

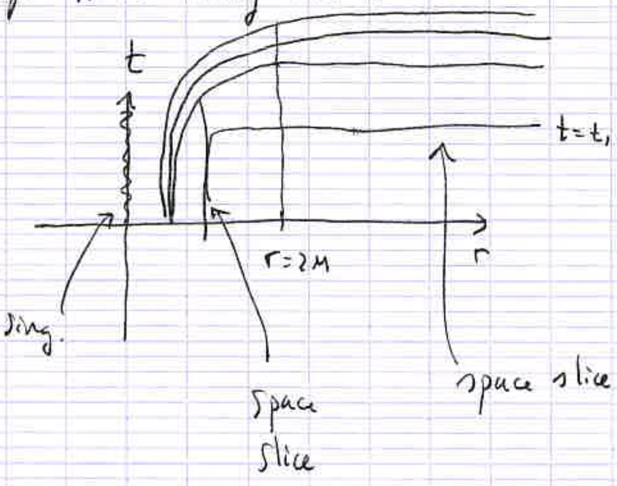


$\omega(t)$ → particle pairs.

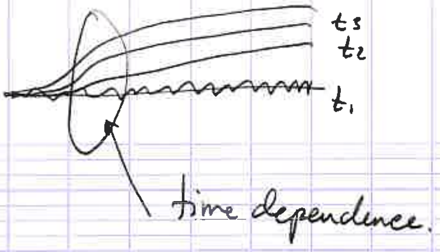
but bh is static so ... why particles.

$$ds^2=-\left(1-\frac{2M}{r}\right)dt^2+\frac{dr^2}{1-\frac{2M}{r}}+r^2d\Omega^2$$

time indep BUT only holds outside



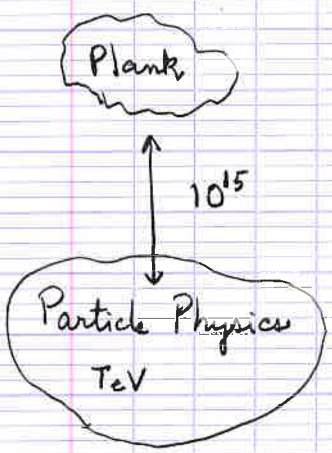
or:



⇒ particle creation.

6/13/05 Open String Phenomenology Verlinde

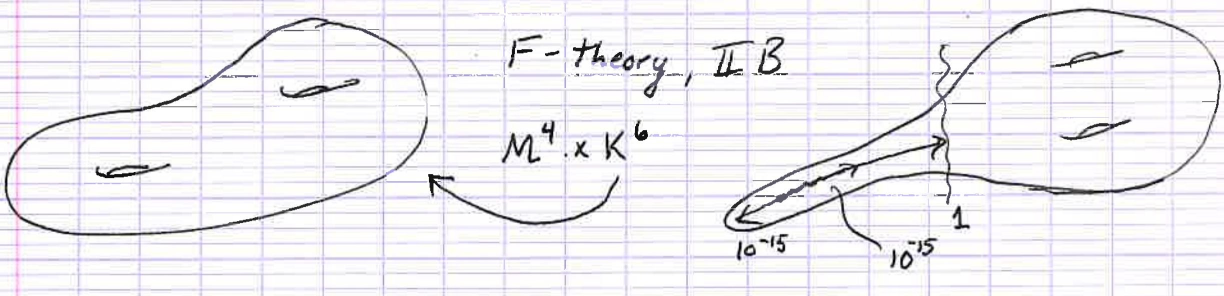
A geometric language for field theory.



∼ Assume LHC does not care about quantum gravity.

"Effective string theory"

Gauge hierarchy ⟷ Geometry (warped)



F-theory, IIB

$M^4 \times K^6$

$$ds^2 = a^2(y)\, g_{\mu\nu}\, dx^\mu dx^\nu + h_{mn}(y)\, dy^m dy^n$$

↑

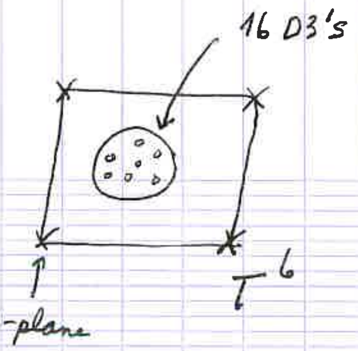
Warp factor

gravitational fields.

scale ⟷ new dimension

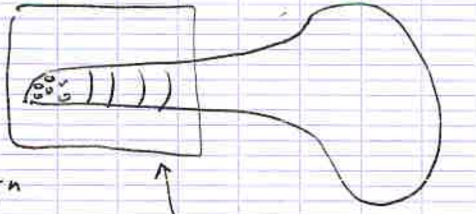
$T^6/_{\mathbb{Z}_2} \times \mathbb{R}^4$ orientifold $\xleftrightarrow[6\times]{\text{T-dual}}$ type I

on $T^6 \times \mathbb{R}^4$

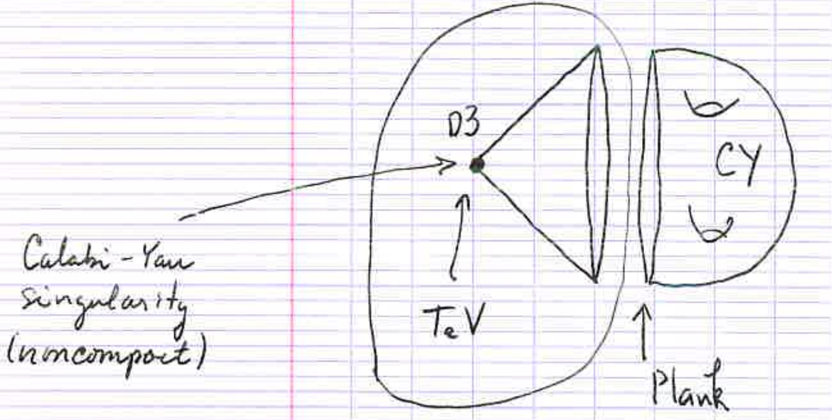


if D3's are confined in a small volume $\rightarrow$ geometry becomes as a warped throat.

$$h_{mn}\, dy^m dy^n = dy^2 + \tilde{h}_{mn}\, d\tilde{y}^m d\tilde{y}^n$$



$$\underbrace{a^2(y)\, g_{\mu\nu}\, dx^\mu dx^\nu + dy^2}_{\approx AdS^5} + \underbrace{\tilde{h}_{mn}\, d\tilde{y}^m d\tilde{y}^n}_{\approx S^5}$$



Decoupling Limit
"get rid of the landscape"

1) $\alpha' \rightarrow 0$ 3) decompactify the region with D3 branes.

2) $M_{Pl} \rightarrow \infty$

$\Rightarrow$ decoupled limit of Open String theory on D3's.

$\rightarrow$ $\boxed{\text{3+1 dim'l quantum field theory}}$

How can we get the standard model?

Relation w/ Kleb. Strassler cascading gauge theory. / holographic RG.

Holographic RG:

M_5 - const. y slices. $\exists\, ds^2(M_5)(y)$.

throat may not be big.

solve classical SUGRA equations. Write them as evolution in y:

$$\boxed{\frac{d}{dy}\phi^I = \beta^I(\phi^I(y))}$$

(c.f. RG evolution of ϕ^I)

we can use this, even in the absence of SUGRA.

closed string
classical SUGRA. $\longleftarrow$ AdS/CFT $\longrightarrow$ RG evolution in QFT

Mathur II

pos. freq.

QFTCS $$\hat{\phi} = \sum_K \frac{1}{\sqrt{V}} \frac{1}{\sqrt{2\omega}} \left(a_K e^{i(Kx-\omega t)} + a_K^\dagger e^{-i(Kx-\omega t)} \right)$$

cc

orthogonal complete

$$a_K|0\rangle = 0$$

$$a_K^\dagger|0\rangle = |1\rangle \text{ etc.}$$

Curved space

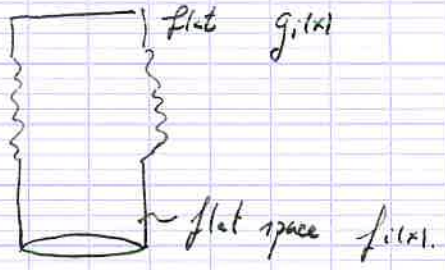
time t

$f_i(x)$ $\quad f_i^*(x)$ $\qquad \hat{\phi}(x) = \sum_i a_i f_i + a_i^\dagger f_i^*$

$\uparrow$

pos freq.

$g_j(x)$ $\quad g_j^*(x)$ $\qquad \hat{\phi}(x) = \sum_j a_i g_i + a_i^\dagger g_i^\dagger$



$a|0\rangle_A = 0$

$b^*|0\rangle_B = 0$

Suppose we take $|0\rangle_A$

$|0\rangle_B$ in terms of $|0\rangle_A$

$$|0\rangle_A = C_0|0\rangle_B + C_1|1\rangle_B + C_2|2\rangle_B + \cdots$$

Schrödinger eqn. $\quad i\frac{\partial\psi}{\partial t} = H\psi \qquad \int dx\, \psi^*(x)\psi(x)$

$\partial_\mu \partial^\mu \phi - m^2\phi^2 = 0 \qquad$ No conserved pos. def inner product.

Klein gordon norm $\rightarrow$ $$i\int_\Sigma d^3x \left[f^* \partial_t g - g^* \partial_t f \right] = (f,g)_{KG}$$

$$(f_K, f_{K'}) = \delta_{KK'}$$

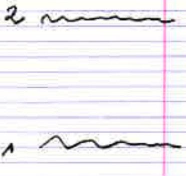
$$(f_K^*, f_{K'}^*) = -\delta_{KK'}$$

not pos. def. but conserved:

$$\partial^\mu [f \partial_\mu g - g \partial_\mu f] = f \Box g - g \Box f = m^2 fg - m^2 fg = 0$$

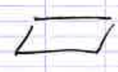
conserved inner product.

$$0 = i\int_1^2 d^4x \sqrt{-g}\, \partial^\mu [f \partial^\mu g - g \partial^\mu f] = \int d^3x$$



$$= \int_2 \cancel{d^3x}\, d\Sigma^\mu [f \partial_\mu g - g \partial_\mu f]$$

$d\Sigma^\mu$



~~$d\Sigma^\mu = \varepsilon^\mu \; dx_1^\mu dx_2^\mu dx^\mu$~~

$d\Sigma^\mu = \varepsilon^\mu{}_{\nu\lambda\sigma}\, dx^\nu_{(1)} dx^\lambda_{(2)} dx^\sigma_{(3)}$

$$- \int_1 \cancel{d^3x}\, d\Sigma^\mu [f \partial_\mu g - g \partial_\mu f]$$

$$(f,g)_{KG} = i\int d\Sigma^\mu \{ f^* \partial_\mu g - g^* \partial_\mu f$$

conserved, nonunitary inner product.

$(f_i, f_j) = \delta_{ij}$

$(f_i, f_j) = -\delta_{ij}$

$(f_i, f_j^*) = 0$

$$(f_K, \underset{\substack{\uparrow \\ \text{expanded in } a\text{'s}}}{\hat{\phi}}) = (f_K, \underset{\nwarrow \text{ expanded in } b\text{'s}}{\hat{\phi}})$$

$$\hat{a}_K = (f_K, g_j)\hat{b}_j + (f_K, g_j^*)\, b_j^+$$

$$\hat{a}_K = \alpha_{Kj}\hat{b}_j + \beta_{Kj}\hat{b}_j^+$$

$$(\alpha_{Kj}\hat{b}_j + \beta_{Kj}\hat{b}_j^+)\,|0\rangle_A = 0$$

$|0\rangle_A = e^{\mu_{ij} b_i^+ b_j^+} |0\rangle_B$ Anzatse

$$b_K = e^{\mu_{ij} b_i^+ b_j^+} |0\rangle_B = 2\mu_{Kj}\, b_j^+ e^{\mu_{ij} b_i^+ b_j^+} |0\rangle_B$$

$\mu_{ij} = ?$

$$\left(\alpha_{kj} \alpha_{jl} 2 \hat{b}_l^+ e^{\mu_{ij} b_i^+ b_j^+} - \beta_{kl} b_l^+ e^{\mu_{ik} b_i^+ b_j^+} \right) |0\rangle_B = 0$$

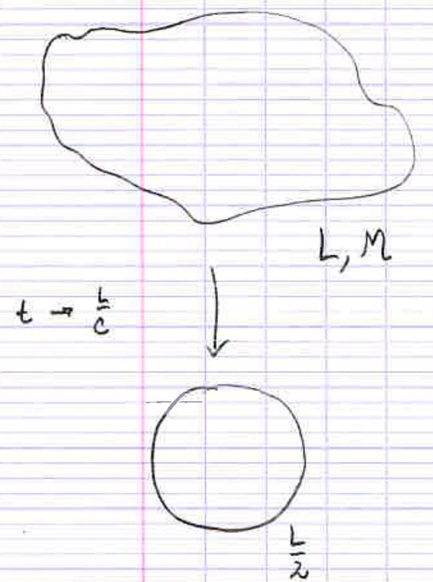
$$2 \alpha_{kj} \mu_{jl} = \beta_{kl} \rightarrow \quad 2\alpha\mu = \beta \qquad \mu = -\frac{1}{2} \alpha^{-1} \beta \qquad \boxed{\mu_{ij} = -\frac{1}{2} \alpha^{-1}{}_{il} \beta_{lj}}$$

$$\Rightarrow \boxed{|0\rangle_A = c\, e^{\mu_{ij} b_i^+ b_j^+} |0\rangle_B}$$

$\alpha_{ij} = (f_i, g_j)$ Bogoliubov transformation.

$\beta_{ij} = -(f_i, g_j^*)$

Physical feel



$g_{ij} = \eta_{ij} + h_{ij}$

$h_{ij} \sim 1$

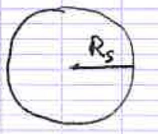
How much particle creation; energies.

$\lambda \sim L$

particles ~ 1 (no big #'s).

Very small energy.

Black hole:



$\lambda \sim R_s$

$\Delta t \sim R_s / c$

$E_{quantum} \sim \frac{\hbar c}{\lambda} = \frac{\hbar c}{R_s}$

$M \sim R_S$ $(R_S = \frac{2GM}{c^2})$ $\qquad \mathcal{E}_{bh} = Mc^2 \qquad$ # quanta $R_S Mc^2/\hbar c$

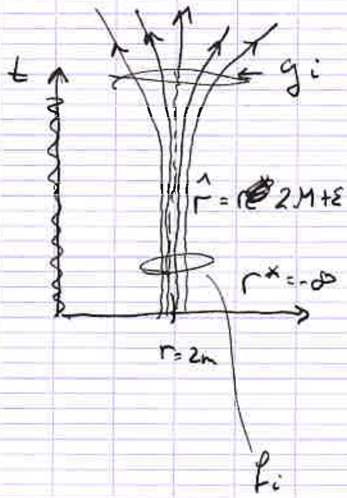
$$T_{evap} \sim \frac{\frac{R_S}{c} \; R_S \frac{c^2}{2G} c^2}{\hbar c} \approx \frac{R_S^2}{M^3} \qquad T_{evap} = t_{plank} \left(\frac{M}{M_{pe}}\right)^3 \sim 10^{63} \text{ yrs.}$$

Fourier Mode → S.H.O.

$\omega = \sqrt{k^2+m^2}$

adiabatic ↦ (VAC → VAC) $\qquad \Delta t \sim \omega^{-1} \Rightarrow$ production of order 1.

Black Hole



Null geodesic $ds^2 = 0$

$dt^2 = dr_*^2$

$t = r^* + c$

$$\Delta t = t_{4M} - t_{2M+\varepsilon}$$
$$= r^*_{4M} - r^*_{2M+\varepsilon}$$
$$= 4M + 4M \log\left(\frac{4M}{2M} - 1\right) - \left[(2M-\varepsilon) + 2M\log(\varepsilon)\right]$$
$$\sim (2M \log(1/\varepsilon))$$

f_i:

$ds^2 = D\, dU\, dV \qquad D = \frac{-32M^3}{r} e^{-r/2M}$

$U = T + X$

$V = T - X$

$ds^2 = [-dT^2 + dX^2]$

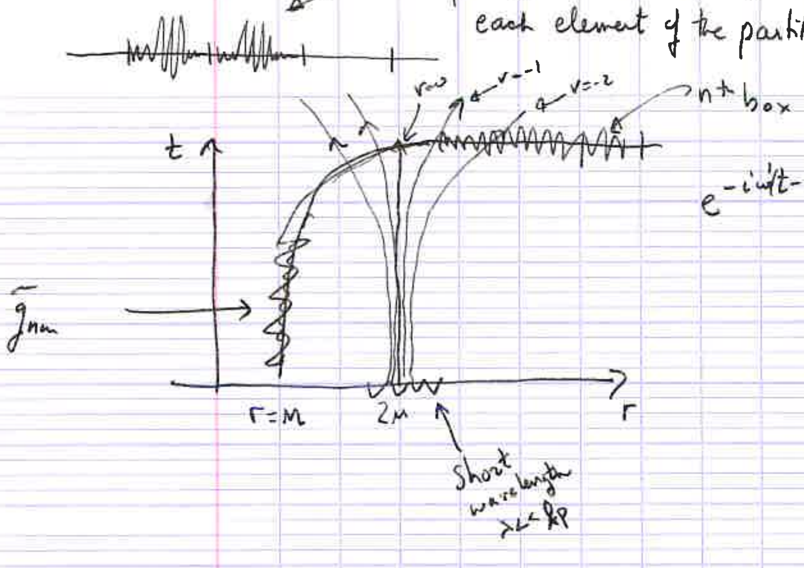
special cases: $\begin{cases} \text{A) flat} \\ \text{B) curved but small region.} \end{cases}$

→ A particle must cost energy.

$|0\rangle_A$ – lowest energy $\qquad$ (say $m=0$)

$a_k |0\rangle_A = |1\rangle$

$$e^{i\omega[X-T]} \sim e^{-i\omega V} \Rightarrow \boxed{f_i = e^{i\omega V}}$$



$$e^{-i\omega'(t-r)} = e^{-i\omega' t}$$

↳ $g_i \to g_{\omega'}$

→ g_{nm} n-th box, m-th box } m-th harmonic

Eikonal approximation

phase of wave function remains constant along a null geodesic

$$(f,g) = \int e^{i\omega V} \overleftrightarrow{\partial_v} e^{-i\omega' u} \qquad \left(V = -e^{-u/4M}\right)$$

$$= \int e^{+i\omega V} \overleftrightarrow{\partial_v} e^{2\omega' 4M \log(-V)}$$

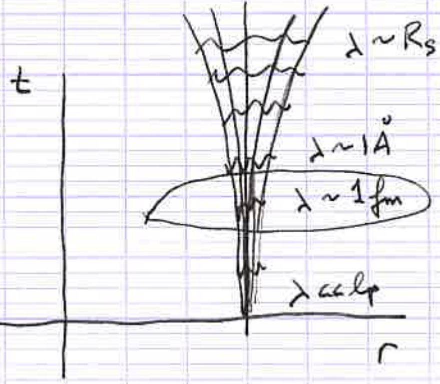
(things are done clearly in Giddings and Nelson PRD 1992 46 2486)

$$|0\rangle_A = e^{\sum_{mn} \mu_{mn} \underbrace{\hat{b}^{+}_{mn} \hat{b}^{+}_{mn}}} |0\rangle_B \Bigg\} \mu_{mn} = -4\pi M \omega_m$$

(s wave dominates)

(do this with coherent states?)

→ Hawking Paradox.



Vac Hamiltonian

↓

Not vac

$|0\rangle + |2\rangle + \ldots$

Nuclear density of matter!?

argument that short wavelength modes don't invalidate

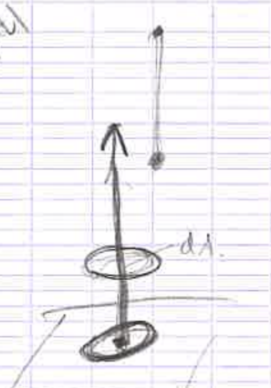
New Hamiltonian? Many States at $E=0$ → $|0\rangle$ not unique.

1) All q-gravity effects ≲ lp or ls
2) Vac unique.

$$\vec{J}(\vec{x},t) = T I \ell \underbrace{\delta(x)\delta(y)\delta(z)}\,\underbrace{\delta(t)}\hat{z}$$

$\vec{J} = \rho\vec{v}$ $\qquad \frac{q}{A}$ $\qquad e^{i\omega t}$

$\int e^{i\omega t}\,d\omega = \delta(t)$



$\vec{dA} = dA\,\hat{z}$

Hertzian Dipole

$$\int \vec{J}\cdot dA = I$$

$$\int_A I\ell\,\delta(x)\delta(y)\delta(z)\delta(t)\,\overset{dxdy}{dA}$$

$$= \begin{cases} 0 & \text{unless } t=0 \\ \end{cases}$$

$[J][A] = \frac{[I]}{t}$

$$\boxed{\vec{J}(\vec{x},t) = (T\ell)\,I\,\delta'_s\,\hat{z}}$$

$[\rho][\vec{v}]$ $\qquad [\rho] \leftarrow \frac{q}{V}$ $\qquad [\vec{v}] \leftarrow \frac{\ell}{t}$

$\vec{J} \propto \underbrace{\delta(t)\,\ell}\,\hat{z}$

$\rho \propto (IT)\,\delta(x)\delta(y)\delta(z)$

$(\vec{J}, \rho)$

$$\cancel{\vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t}}$$

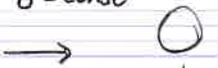
$$TI\ell\,\delta(x)\delta(y)\delta'(z) + \cdots = -\frac{\partial}{\partial t}\,IT\,\delta(x)\cdots = 0$$

Scratch:

$\vec{E} = \text{const}$

x(t) unbounded soln.

$\vec{B} = \text{const}$



x(t) = bounded soln.

E → sources.
B → no sources.

AdS

---

hep-th/0505044 D. Marolf.

BH instability in 5D?

3+1	parameter	Bounds	uniqueness	Stability
Schwarzschild	M	M > 0	✓	✓
Reisner Nordstrom	M, Q	M ≥ Q	✓	✓
Kerr	M, J	$M^2 \geq J$	✓	✓ hard.

OR
- naked sing
- CTC's

4+1

many (nonunique) solution : given M, J, ∃ more than 1 soln.

→ BH : S^3

→ B-rings (2001): $S^1 \times S^2$

can be unstable.

SUSY:

1 Supertubes:
- BPS 1/4 SUSY.
- in IIA
- 2 charges F1 & D0 → q_{F1} q_{D0}

$\vec{E}\times\vec{B}\neq 0 \Rightarrow \exists\ j$: angular momentum.

$\boxed{ST \mid q_{D0},\ q_{F1},\ j}$

WORDVOLUME

Probes

2. BMPV Black hole

IIB D1-D5-P $M^5 \times S^1 \times T^4$ } spin

	M^5	S^1	T^4
D1		×	
D5		×	
	×		

$\Rightarrow$ BH w/angular momenta $\left[\frac{1}{4}\text{ SUSY, BPS}\right]$.

IIA: T dualize along T

D0-D4-F1 $J_{12} = J_{34} = J \quad J^2 \leq Q_{D0} \times Q_{F1} \times Q_P$

3. Cvetic - Youm Black holes.

- non-BPS
- same charges: Q_{D0}, Q_{D4}, Q_{F1}, energy: δE
- $J_1 \neq J_2$ but $J_1 - J_2 \leq \delta E$

$$\left(\frac{J_1+J_2}{2}\right)^2 \leq Q_1 Q_2 Q_3 (1 + c\,\delta E)$$

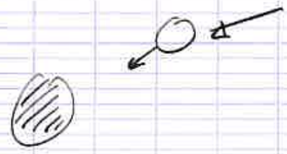
Supertube + BMPV → BPS (Bena & Kraus)

$j \qquad J_1 > J_2 \longrightarrow J_1 + j_1 \,;\, \underset{\otimes}{J_2} \longrightarrow$ SUSY black rings.

$S_{BR} < S_{BMPV}$

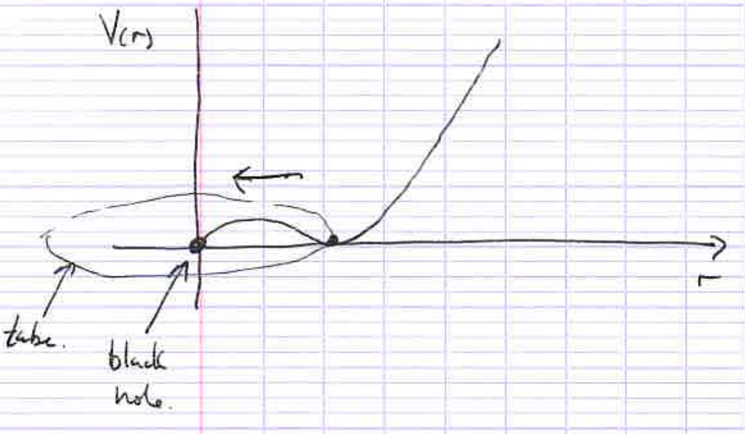
$ST + BMPV + \delta E \longrightarrow \qquad \frac{\delta E}{M_{BMPV}} \ll 1$

↑ ST can move



ST's are loops that can move.

DBI → KE + PE



$$\left.\begin{array}{l} Q_{D0} + q_{D0} \\ Q_{F1} + q_{F1} \\ Q_{D4} \\ J_1 + j_e \\ J_2 \\ \delta E \end{array}\right\}$$

State obeys none of Cvetic - Youm bounds.

Tchani Finch.
Chronology Protection
in BMPV BH

Dyson hep-th/0302052

$$ds^2 = -(f_1 f_5 f_K)^{-2/3}\left[dt^2 + \frac{J}{2r^2}(\sin^2\theta\, d\phi_1 - \cos^2\theta\, d\phi_2)\right]^2$$

$$+ (f_1 f_5 f_K)^{1/3}\left[dr^2 + r^2 d\theta^2 + r^2\sin^2\theta\, d\phi_1^2 + \cos^2\theta\, d\phi_2^2\right]$$

$$A = \frac{2\pi^2}{G_5}\sqrt{Q_1 Q_5 Q_{KK} - \frac{J^2}{4}} = 4 S_{BH}$$

$$\tau = T_0{}^0 = \frac{2}{r^3}\left(Q_{KK} - \frac{J^2}{4(Q_1 + r^2)(Q_5 + r^2)}\right)$$

$$R_{cp} = \frac{Q_1 + Q_5}{2}\left[-1 + \sqrt{1 - \frac{4}{Q_{KK}(Q_1 + Q_5)^2}\left(Q_1 Q_5 Q_{KK} - \frac{J^2}{4}\right)}\right]$$

Rotating & SUSY only in 5D (?).

IIA D_4 D_0 F1 $AdS_2 \times S^3$ near horizon

IIB D_5 D_1 P_5 ← KK particles.

$T^4 \times S^1 \times M^5$

(Kerr and CTC's)? $J \leq M^2$ (if it is violated CTC's get naked).

BMPV - hep-th/960265

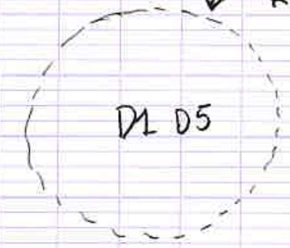
$$4(r^2+Q_1)(r^2+Q_5)(r^2+Q_{KK}) < J^2$$

$$J^2 > 4Q_1Q_5Q_{KK}$$

String theory resolution of this. I.e.

Treat D_5 D_1 KK → as probes. Bring them in from ∞.

↙ KK's can be brought in beyond the bound.



$$J^2 > 4Q_1Q_5Q_{KK}$$

D1/D5/KK outside $t, r, \theta, \phi_1, \phi_2, z$.

D1/D5 inside $t', r, \theta, \theta', \phi_1', \phi_2'$

Domain Wall

$T_{\mu\nu} \neq 0$ (see $T^0_0 = c$).

$T^0_0 = 0 \Rightarrow R_{cp}$

Klaus hep-th/0502111

$$ds^2 = -\left(1+\frac{r^2}{R^2}-\frac{r_h^2}{r^2}\right)dt^2 + \frac{dr^2}{(\cdots)} + r^2(d\theta^2 + \sin^2\theta\, d\Omega_2^2)$$

$|\Psi\rangle = \psi_1|m_1\rangle + \psi_2|m_2\rangle$ Coral grav field. $\phi =$

$m_1 + m_2$

$\downarrow$

e $H = \frac{p_1^2}{2m_1} + \frac{p_1^2}{2m_1} + V_e(\phi)$

$$V_e\phi = G\,\frac{Mm_1}{\cdot}$$

de Alwis I :

Effective potentials for light moduli

- Wess/Bagger
- Gates, Grisaru ....
  2001

* Review $N=1$ SUGRA
  -F , -D Pot'l

* Weyl Anomalies generate nonperturbative terms in W

* Integrating out heavy fields.
  a) global SUSY
  b) SUGRA

* Application of KKLT.

Global SUSY :

$\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2i\sigma^m_{\alpha\dot\alpha}\partial_m$ $\qquad Q_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^m_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_m$

$\{Q_\alpha, Q_\beta\} = \cdots$ $\qquad \bar{Q}_{\dot\beta} = \underline{\qquad}$

$\{D_\alpha, \bar{D}_{\dot\alpha}\} = -2i\sigma^m_{\alpha\dot\alpha}\partial_m$ $\qquad D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^m_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_m$

$\{D_\alpha, D_\beta\} = \cdots = 0$

$\{D_\alpha, Q_\beta\} = \cdots = 0$

Chiral Superfields

$$\bar{D}_{\dot\alpha}\Phi(x,\theta) = 0 \rightarrow \Phi(x,\theta) = \{A(x), \psi_\alpha(x), F(x)\}$$

Actions: $S = \underbrace{\int d^4_x d^4\theta\, K(\phi,\bar\phi)}_{\text{Kahlerpotential}} + \int d^4_x d^2\theta\, \underset{\uparrow \text{ superpotential}}{W(\Phi)} + \int d^4_x d^2\bar\theta\, \bar{W}(\bar\Phi)$

$\int d^2\theta \rightarrow -\frac{1}{4}D^\alpha D_\alpha \equiv -\frac{1}{4}D^2 \qquad d^4\theta = d^2\theta\, d^2\bar\theta = \frac{1}{16}D^2\bar{D}^2$

Kahler transf. $K \rightarrow K(\Phi,\bar\Phi) + f(\Phi) + \bar{f}(\bar\Phi)$

Real Superfield $V = \bar{V}$ $\quad V(C, \chi^\alpha, \bar{\chi}^{\dot\alpha}, M, N, \lambda^\alpha, \bar{\lambda}_{\dot\alpha}, V_m, D)$

$$D \xmapsto{\text{SUSY}} \text{total derivatives}$$

$$F \xmapsto{\text{SUSY}} \quad ''$$

$$A = \Phi\Big|_{\theta\bar\theta=0} \qquad \psi = \frac{1}{\sqrt{2}} D_\alpha \Phi\Big|_{\theta=\bar\theta=0} \qquad F = -\frac{1}{4} D^2 \Phi$$

$$\mathcal{L} = -V = F\bar{F} + Ff(A) + \bar{F}\bar{f}(\bar{A})$$

$$\bar{F} = -f(A)$$

$$-V = |f|^2 - |f|^2 - |f|^2 = -|f|^2$$

$$S = \int d^4x\, d^2\theta \left[-\frac{1}{4}\bar{D}^2 K(\Phi, \bar\Phi) + W(\Phi) \right] + \int d^2\bar\theta\, \bar{W}(\bar\Phi)$$

$$\delta_\Phi S = \int d^4x\, d^2\theta \left[-\frac{1}{4}\bar{D}^2 K_i(\Phi, \bar\Phi) + W_i(\Phi) \right] \delta\Phi$$

$$K_i = \frac{\partial K}{\partial \Phi^i} \qquad \frac{1}{4} K_{i\bar{j}} \bar{D}^2 \bar\Phi^{\bar{j}} + K_{i\bar{j}\bar{\ell}} \bar{D}_{\dot\alpha} \bar\Phi^{\bar{j}} \bar\Phi^{\bar{\ell}} = \frac{\partial W}{\partial \Phi^i}$$

$$\bar{F}^{\bar{j}} - K^{i\bar{j}}_{\cancel{eg}} \partial_i W + \text{ferm} \ldots$$

$$\underbrace{f(x,y)} \rightarrow \frac{\partial^2 f}{\partial x^i \partial x^j} \geq 0 \Rightarrow \underbrace{\text{no negative e'vals.}}$$

$x^1 = x \quad x^2 = y$

$$\searrow M_{ij} v^i w^j \geq 0$$

Verlinde.

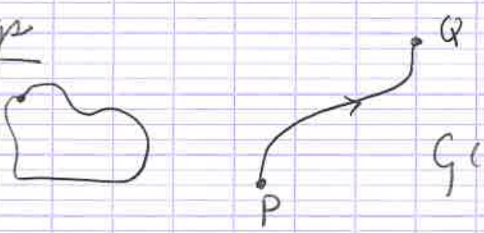
$$\begin{tikzcd}
\text{D3 on CY singularity} \arrow[d, Rightarrow] \arrow[dd, Rightarrow, bend left=60] \\
\text{Quiver gauge theory} \arrow[d, Rightarrow, "\text{symmetry breaking}"] \\
\text{MSSM}
\end{tikzcd}$$

G a finite subgrp. of $SU(3)$ ($\Rightarrow$ $N=1$ gauge thy).

$\mathbb{C}^3/G$

$G \to$ quiver

Closed strings



$G(P) = Q$ identified.

G copies of D3-brane

$U(|G|)$ gauge theory $N = |G|$ order

$$\left.\begin{array}{l} V = \text{gauge multiplets} \\ \Phi^I = \text{chiral multiplets (3)} \end{array}\right\} \text{adjoint of } G$$

$R_{reg} V R_{reg}^{-1} = V$ $\quad$ R_{reg} = regular rep. of G

$R_{3\,IJ} R_{reg} \Phi^I R_{reg}^{-1} = \Phi^J$

$$\boxed{\begin{array}{l} \mathbb{C}[G] = \left\{ \sum_{\gamma\in G} x(\gamma)\,\gamma \right\} \\ \text{group algebra} \end{array}}$$
$$\Uparrow_N$$

G acts on $x \in \mathbb{C}[G]$ by

$g \in G \qquad gx = \sum_{\gamma\in G} x(g^{-1}\gamma)\,\gamma \Rightarrow N\times N$ matrix representation of G

$$R_{reg} = \bigoplus_{a=1}^{r} n_a R_a \qquad \Bigg| \qquad \text{Thm: } N = \sum_{a=1}^{r} n_a^2$$

irreps $a = 1, \dots, r$

$n_a = \dim R_a$

$$R_{reg} = \begin{pmatrix} R_1 \otimes \mathbb{1}_{n_1} & & & 0 \\ & R_2 \otimes \mathbb{1}_{n_2} & & \\ & & \ddots & \\ 0 & & & R_r \otimes \mathbb{1}_{n_r} \end{pmatrix}$$

$$R^a \otimes \mathbb{1}_{n_a} = \left.\begin{pmatrix} R_a & & \\ & R_a & \\ & & \ddots \; R_a \end{pmatrix}\right\} n_a$$

$$U(N) \rightarrow \prod_{i=1}^{r} U(n_a) \quad \text{symmetry breaking}$$

Terminology:

$$\begin{tikzcd}
\text{Fractional brane} \arrow[r, leftrightarrow] \arrow[d, Leftrightarrow, "1-1"] & U(n_a) \\
\text{Irreps. of } G \arrow[d, Leftrightarrow, "1-1"] & \text{multiplicity } n_a \arrow[ul, bend left] \\
\text{conjugacy classes of } G.
\end{tikzcd}$$

$U(n_a) \leftarrow$ nodes of quiver

lines between $\textcircled{n_a} \longrightarrow \textcircled{n_b}$

bi-fundamental fields $(n_a, \bar{n}_b)$

~~$R_3 R_{reg}$~~ $R_3 R^a = \boxed{n^3_{ab}} R^b$

$\uparrow$ how many lines there are

$$\boxed{R_{reg} = \bigoplus_{a=1}^{r} n_a R_a}$$

<u>Dual SUGRA</u>:

$$AdS_5 \times S^5/G \xleftrightarrow{\text{dual}} \text{quiver gauge theory.}$$

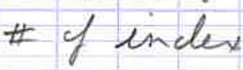
If we assume AdS/CFT works, we can prove above duality

Ex: Open closed string duality $\mathbb{C}^0/q$

$$\int dZ\, e^{-\frac{1}{2} N t_2 Z^2}\, e^{N \sum_{j\geq 3} t_j \mathrm{Tr}\, Z^j} = Z_N(t)$$

$Z = N \times N$ matrix $\Uparrow$ Feynman diagram.

't Hooft



$V(\Gamma) = \#$ vertices $= \sum_j v_j(\Gamma)$

$P(\Gamma) = \#$ propagators

$f(\Gamma) = \#$ of index loops (faces)

Euler: $\chi(\Gamma) = 2 - 2g = V(\Gamma) - P(\Gamma) + f(\Gamma)$

$Z_N(t)$ can be expanded in powers of ~~the~~ g

$$\log Z_N(t) = \sum_{g\geq 0}' \frac{1}{N^{2g-2}} F_g(t)$$

χ <-> # closed string loops <-> genus (g)

$$\left.\begin{array}{l} t \to t_{crit} \\ N \to \infty \end{array}\right\} \to \text{string expansion}$$

$\boxed{c=0 \text{ noncritical string theory}}$ (has D-branes)

Let's <u>mod out G</u>: $(N=1)$

$$\ln Z_G(t) = \sum_{g\geq 0}' \tilde{N}^{2-2g} \tilde{F}_g(t)$$

$$\tilde{N} = |G| N \qquad \tilde{F}_g(t) = Z_g(G) F_g(t)$$

$$\boxed{\tilde{Z}_g(t) = \sum_a (S_a)^{2-2g}} \leftarrow \boxed{S_a = |G| / \dim R_a}$$

pf: a) open string 1st:

$\underline{\underline{V}}$ – matrix variables $|G| \times |G|$

~~&~~ $V \in \mathbb{C}[G] = \bigoplus_a \underbrace{\mathrm{Mat}(n_a)}_{U(n_a) \text{ acts on it}}$

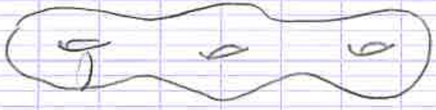
$$Z_{\text{open string}}(G) = \int dV\, e^{\tilde{N} t_2 V^2 + \Sigma' \cdots}$$

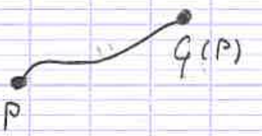
$$= \prod_{a=1}^{r} Z_{n_a N}(t)$$ product of partition fns of "fractional branes"

Now take ln.

b) Closed string calculation:

Let's look at genus g

$\Sigma \sim$ 



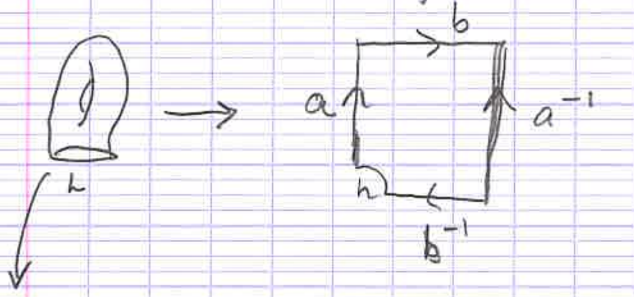
(orbifold cohomology)

• (Σ' twisted sectors)

project out G-invariant states ↔ $\frac{\text{Hom}(\pi^1(\Sigma), G)}{G}$

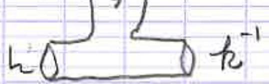
# of elements in $\text{Hom}(\pi^1(\Sigma), G)$

So, what is the order of $\text{Hom}(\pi^1(\Sigma), G)$

g=1 

$aba^{-1}b^{-1} = h$

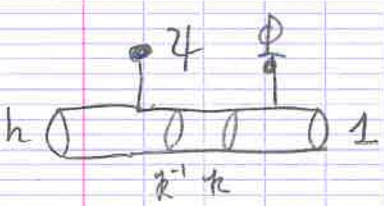
$$Z_1(h) = \sum_{a,b} \delta(aba^{-1}b^{-1} - h)$$

$$Z_1(k^{-1}h) =$$ 

Start gluing:

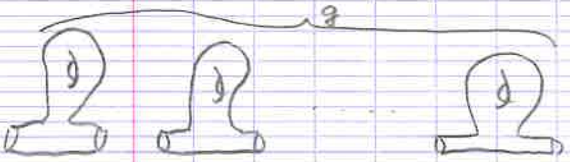
$$(\psi \bullet \phi)(h) = \sum_k{}' \psi(hk^{-1})\phi(k)$$

(gluing product)



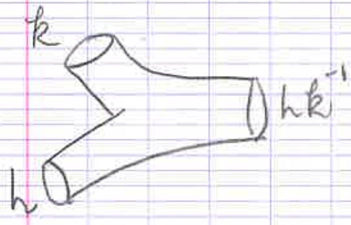
[Also think: counting twisted boundary conditions]

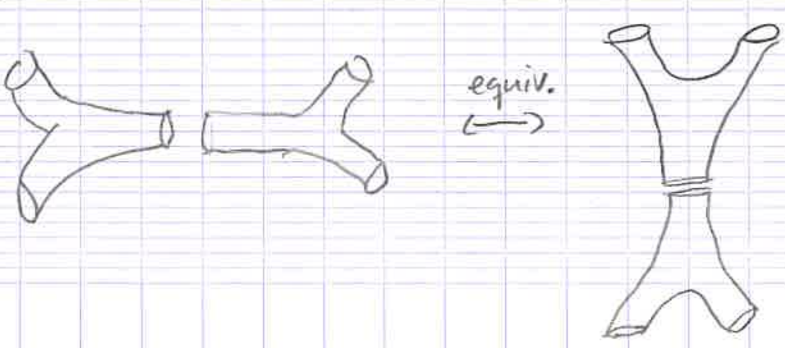
$$\psi_g(h) = (\psi_1 \bullet \psi_{g-1})(h)$$



$$\psi_g(1) = Z_g(t)$$
$$\parallel$$
$$\frac{|\mathrm{Hom}(\pi'(\Sigma), g)|}{|G|}$$

Why it's commutative





commutativity follows from

~~ψ's are in algebra : $(\mathbb{C}^0/G)$~~

basis of class functions ~~ψ_a~~ $\chi_a(h) = tr_{R_a}(h)$ (character.)

$$\frac{1}{|G|}\sum_g \chi_a(g)\chi_b(g) = \delta_{ab}$$ completeness & orthogonality

$$\delta(z-1) = \sum_{a=1}^{r} n_a \chi_a(z)$$

decompose ψ's in terms of group characters.

$$\psi_1(h) = \sum_{a=1}^{r} S_a \chi_a(h)$$

$$S_a = \frac{|G|}{n_a}$$

$$\cancel{\sum_{h\in G}\chi_a(h)} \quad \sum_{h\in G}{}' \chi_a(uhvh^{-1}) = S_a\chi_a(u)\chi_a(v)$$

χ's behave nicely under gluing product

$$\boxed{\chi_a \cdot \chi_b = \delta_{ab} S_a \chi_a}$$

$$\cancel{Z} \quad \tilde{Z}_g(G) = \sum_{a=1}^{r}{}' (S_a)^{2-2g}$$

||OPEN-CLOSED||

Baby example of open/closed duality

Quiver gauge theory on $\mathbb{C}^3/\Delta_{27}$

$$A_{i,j} = \begin{pmatrix} \omega^i & & \\ & \omega^j & \\ & & \omega^{-i-j} \end{pmatrix}$$

$$\omega = e^{2\pi i/3}$$

$$C_{i,j} = \begin{pmatrix} 0 & 0 & \omega^i \\ \omega^i & 0 & 0 \\ 0 & \omega^{i-j} & 0 \end{pmatrix}$$

$$E_{i,j} = \begin{pmatrix} 0 & \omega^i & 0 \\ 0 & 0 & \omega^j \\ \omega^{i-j} & 0 & 0 \end{pmatrix}$$

$$\Delta_{27}$$

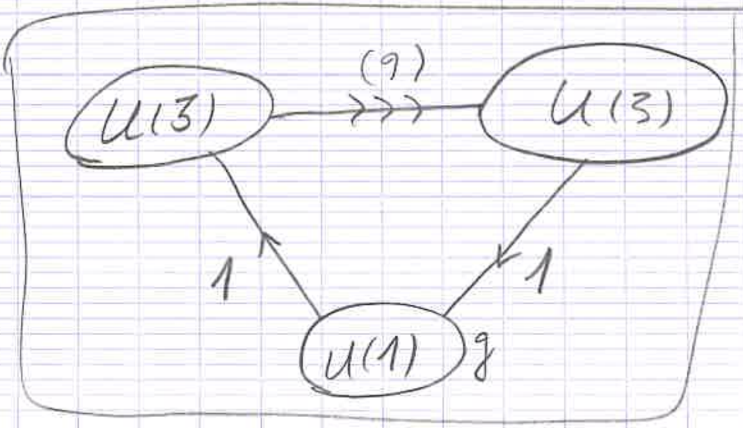
$$\underbrace{(\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes \mathbb{Z}_3}_{27 \text{ elements}}$$

R_a

$n_a = 1$

$n_3^1 = 3$

$n_3^2 = 3$



Berenstein et. al.

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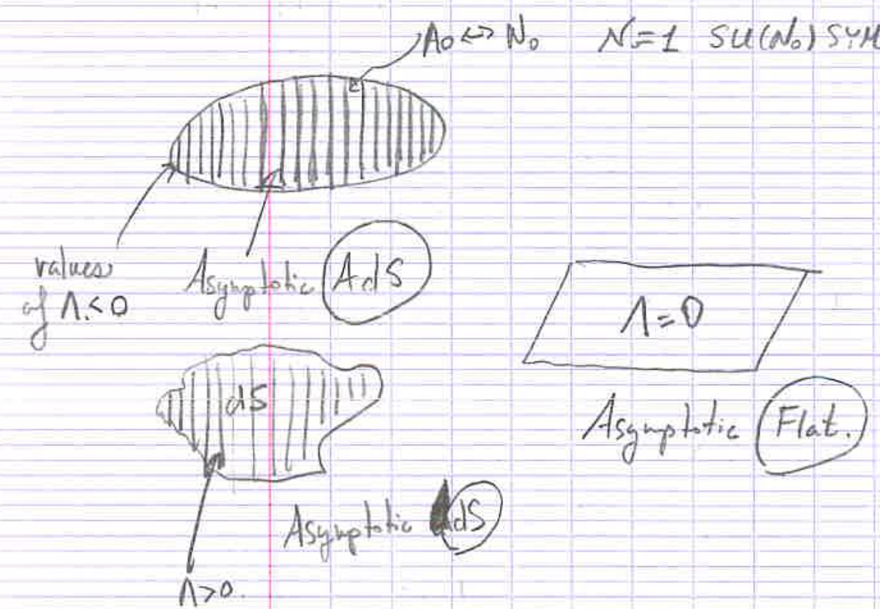
Some Random thoughts:

What is SUSY?

$N=1$ $SU(N_0)$ SYM theory



c.f. Gelfand

Ed Can one ~~find~~ study the representations of the Diffeomorphism group motivated by classification by asymptotic boundary conditions?

What are possible asymptotic BC's in GR?

$$g \to \left\{ \begin{array}{l} \to \text{AdS} \\ \to \text{dS} \\ \to \text{FLAT} \end{array} \right\}$$

maximally symmetric ⇒ max # of Killing vectors $= n_{max}$

What about

$g \to$ {
- space with 1 less Killing vector $\underline{n_{max}-1}$
- What spaces are these?

remove 1 KV:

g

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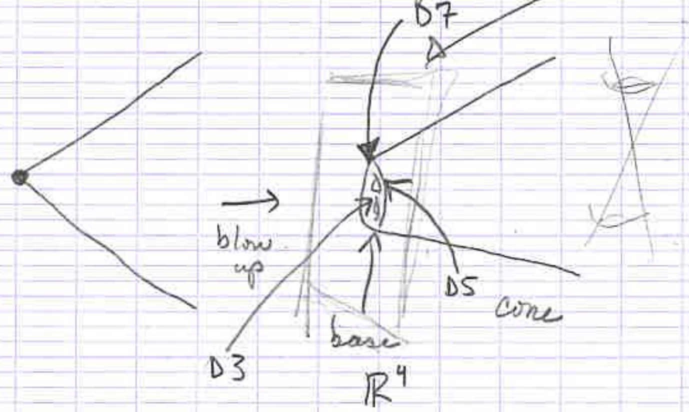
Verlinde III

Kontsevich - D-branes in curved space.



everything happens down here

del Pezzo surfaces (singularities) Base dP_n $\longrightarrow$ homology

$\dim_{\mathbb{C}} = 2$

$\boxed{1}$ ← 4 cycle : D7

$\boxed{n+1}$ 2 cycles : D5

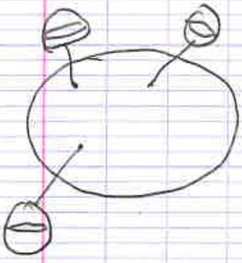
$\boxed{1}$ 0 cycle : D3

Fractional branes: (blow up construction of previous lecture)

1 fractional brane per representation of G.

$\#$ Fractional branes $= n+3 = (n+1)+1+1$

$ch(F_i) = (\underset{\text{rank}}{D7}, \underset{\text{mag. flux}}{D5}, \underset{\text{instanton \#}}{D3})_{\text{charge}}$ charge vector. $= (rk(F_i), c_1(F_i), ch_2(F_i))$ [configuration will have $N=1$ SUSY]



$\mathbb{P}^2$ Blow up n points to n S^2's.

each blow up points will have

$E_i =$ "exceptional divisor" $i = 1, \dots, n$

$H =$ hyperplane class of $\mathbb{P}^2$

(del Pezzo is toric for n small non toric n large) intersection numbers # (where D-branes intersect → light strings)

$$\left.\begin{aligned} E_i \cdot E_j &= -\delta_{ij} \\ H \cdot H &= 1 \\ H \cdot E_i &= 0 \end{aligned}\right\} \text{ intersection pairing}$$

<u>Canonical Class</u> – (tells tangent bundle of del Pezzo)

$$K = -3H + \sum_{i=1}^{n} E_i \qquad K \cdot K = 9-n$$

Single D3 brane near singularity

$\|$ equiv.

collection of fractional branes F_i with multiplicity n_i

$$\boxed{\begin{aligned} &\sum_i n_i \, ch(F_i) = (0,0,1) \\ &n_i \to U(|n_i|) \end{aligned}}$$

Why numbers are negative?

some will be negative.

intersection from 6D perspective

$\#(F_i, F_j)$ intersection number = # of lines between nodes $U(n_i)$ & $U(n_j)$

↓

$\chi(F_i, F_j)$

Euler number.

$$\chi(F_i, F_j) = rk(F_i)\deg(F_j) - rk(F_j)\deg(F_i)$$

$$\deg(F_i) = -K \cdot c_1(F_i)$$

= # intersection points between $c_1(F_i)$ & 4-cycle

Fractional branes ↦ Quiver Diagram

$\boxed{dP_1}$ Example: H, E_1

4 fractional branes F_i

$\boxed{n_i = -1}$

$ch(F_1) = (2, -H, -\frac{1}{2})$

$ch(F_2) = (0, E_i, -\frac{1}{2})$

$ch(F_3) = (-1, H - E_i, 0)$

$ch(F_4) = (-1, 0, 0)$

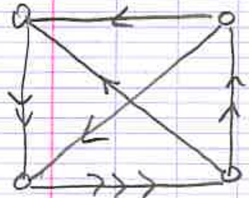
Basis of fractional branes are called "collections"

$$\chi(F_i, F_j) = rk(F_i) deg(F_j) - rk(F_j) deg(F_i)$$

$\Uparrow$

Best understood (stable) collections are "exceptional"

- no adjoint matter
- $\#(F_i, F_j) = 1$ gauge multiplets



Quiver

Can we get standard model?

(c.f. ADE classification)

del Pezzo 8 ↔ E_8 connection

2 cycles: $H, E_i \longrightarrow K = -3H + \sum_{i=1}^{8}{}' E_i$

$$\alpha_i = E_{i+1} - E_i \quad i = 1, \dots, 7$$

$$\alpha_8 = H - E_1 - E_2 - E_3$$

$\alpha_i \cdot \alpha_j = -A_{ij}$ cartan matrix of E_8

Mathematicians: (some mistakes)

$$ch(F_i) = (1, H - E_i, 0) \quad i = 1, \dots, 4$$

$$ch(F_i) = (1, -K + E_i, 1) \quad i = 5, \dots, 8$$

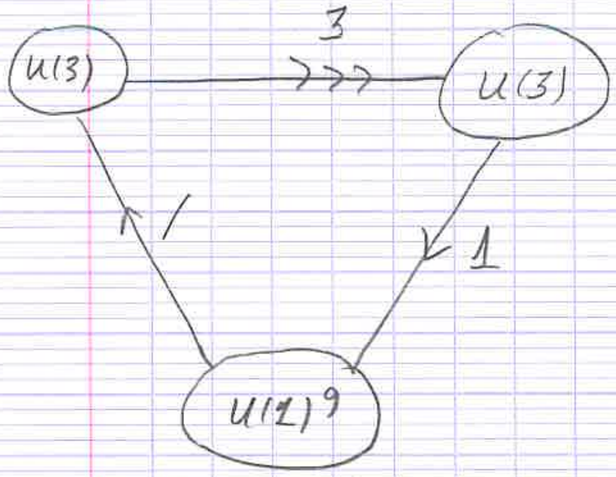
$$ch(\underset{\substack{\uparrow \\ F_9}}{F_9}) = (1, 2H - \sum_{i=1}^{4} E_i, 0)$$

$$ch(F_{10}) = (1, E_4, -1/2)$$

$$ch(F_{11}) = (4, -2K + {\sum_{i=4}^{8}}' E_i, 1/2)$$

$$-n_{10} = n_{11} = -3 \quad n_i = 1 \quad i = 1, \dots, 9$$

Quiver diagram:



<u>more on del Pezzo 8</u>

hypersurface of degree 6 in a weighted projective space (~~pr~~ weighted projective space = invariant under weighted rescaling)

$WP^3_{1,1,2,3}$ (x, y, z, w)

$w^2 = Az^3 + By^6 + Cx^6 + \cdots$ ← 8 parameters

Δ_{27}: $(X, Y, Z) \longmapsto (g_1, g_2, g_3)$

$$g_1 = (e^{\frac{2\pi i}{3}} X, e^{-\frac{2\pi i}{3}} Y, Z)$$

$$g_2 = (X, e^{\frac{2\pi i}{3}} Y, e^{-\frac{2\pi i}{3}} Z)$$

$$g_3 = (X, Y, Z)$$

invariants
$XYZ = x$
$X^3 + Y^3 + Z^3 = z$
$(X^3 + w Y^3 + w^2 Z^2)(X^3 + w^2 Y^3 + w Z^3) = y$
$(X^3 + w Y^3 + w^2 Z^3)^3 = w$

weights $(x, z, y, w) = (1, 1, 2, 3)$

$$\boxed{w^2 + + y^3 - 27 w x^3 + w z^3 - 3wxyz = 0}$$

→ cubic superpotential in this theory with 27 superpotential terms (Yukawa couplings)

Field redefinitions $GL(3) \times [GL(1)]^9$ + 1 overall

$27 - 9 - 9 + 1 = 10$

(Mirror symmetry of del Pezzo's)

Geometric Dictionary:

In principle, ∃ 1-1 correspondence between the space of all deformations of D3 gauge theory ~~[illegible]~~ and closed string geometry fields.

- superpotential $\overset{1-1}{\longleftrightarrow}$ complex structure deformations

- gauge couplings:

$$\tau_I = \theta_I + i\,\frac{8\pi^2}{g_I^2} = \int_{C_I \leftarrow 2\,\text{cycle}} (C^2_{RR} - \tau B^{NS})$$

J - Kähler form. (4-form) $*J \leftarrow 2$ form

$$\Rightarrow \int_{C_I} *J \leftarrow \text{FI parameter (blow up parameter)}$$

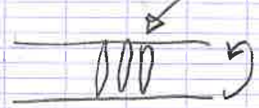
6/16/05

Mathur II

Black Holes in String Theory

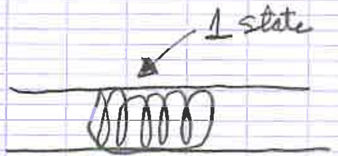
II A

NS1 brane



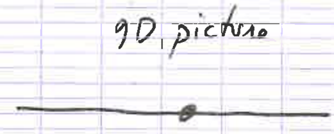
$y \sim y + 2\pi R$ want to make these bound states

$M_{9,1} \rightarrow M_{8,1} \times S^1$



heavily wrapped string.

$n_1 >> 1$



$$ds^2_{string} = H_1^{-1}[-dt^2 + dy^2] + \sum_{i=1}^{8} dx^i dx_i \qquad e^{2\phi} = H_1^{-1}$$

$$H_1 = 1 + \frac{Q_1}{r^6}$$

$\Rightarrow \boxed{A_H = 0}$ $\quad \frac{A^E_{10}}{4G_{10}} = ?$ OR $\frac{A^E_9}{4G_9} = ?$

Actually $\frac{A_{10}}{4G_{10}} = \frac{A_9}{4G_9}$ cause $G_9 = \frac{G_{10}}{2\pi R}$

Einstein Frame $G_E = G_s = e^{-\phi/2}$

$\Rightarrow S_{BEK} = 0$ which is OK because $= \ln[1]$

$\frac{A}{4G} = S_{BEK}$

state $e^{S_{BEK}}$

but... no hair $\Rightarrow \ln[\#\text{states}] = 0$

for $M = M_\odot \Rightarrow 10^{10^{77}}$ states

As $H_1^{-1} \xrightarrow[\phi\to\infty]{} 0$ M-thy

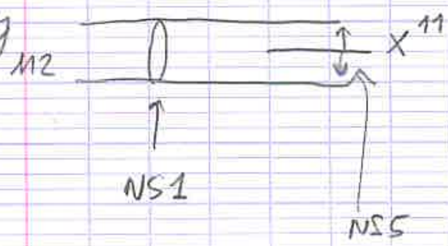
$x_{11} \to 0$

along brane shrinks → tension.
others expand. → flux lines want to repel

but $\frac{A_{11}}{4G_{11}} = 0$ is obvious

To balance it out.. take

M-theory (256 degeneracy)



$M_{9,1} \to M_{4,1} \times S^1 \times T^4$

NS1 NS5

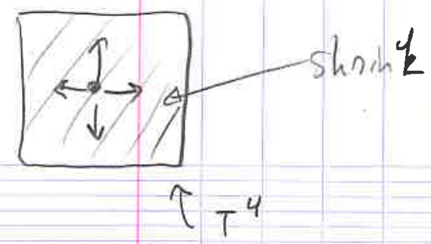
noncompact

$$ds^2_{\text{string}} = H_1^{-1}[-dt^2 + dy^2] + H_5 \sum_{i=1}^{4} dx^i dx^i + \sum_{a=1}^{4} dz_a dz_a$$

T^4

$e^{2\phi} = \frac{H_5}{H_1}$ → finite

$H_1 = 1 + \frac{Q_1}{r^2}$ $H_5 = 1 + \frac{Q_5}{r^2}$



$\Rightarrow$ T^4 stabilizer.

$$\overset{x,t}{M_{4,1}} \times \overset{y}{S^1} \times \overset{z_a}{T^4}$$

y is now shrinking!

Try to stabilize y:

momentum from any massless field.

$\frac{2\pi n_p}{L}$ — momentum modes try to expand y.

TL — brane.

$$\Rightarrow \quad ds^2_{String} = H_1^{-1}\left[-dt^2 + dy^2 + K(dt \cancel{+} dy)^2\right] + H_5 \sum_{i=1}^{4} dx_i dx^i$$

$$+ \sum_a dz_a dz_a$$

$$e^{2\phi} = \frac{H_5}{H_1} \qquad H_1 = 1 + \frac{\overset{n_1}{Q_1}}{r^2}, \quad H_5 = 1 + \frac{\overset{n_5}{Q_5}}{r^2} \quad K = \frac{\overset{n_p}{Q_P}}{r^2}$$

$r \to 0$

$$H_5 = \left[dr^2 + r^2 d\Omega_3^2\right] \rightarrow \frac{Q_5}{r^2}\left[dr^2 + r^2 d\Omega_3^2\right]$$

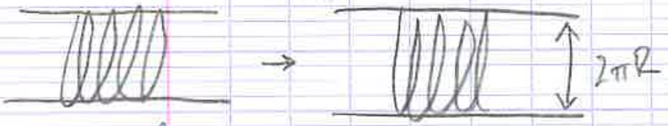
$$\hookrightarrow Q_5 d\Omega_3^2$$

$$A_{String} = \underset{\substack{\uparrow \\ \text{surrounding} \\ M_{4,1}}}{[S^3]}[T^4][S^1] \qquad \begin{array}{l} S^1 = 2\pi R \text{ at } \infty. \\ T^4 = (2\pi)^4 V \text{ at } \infty \end{array}$$

$$\frac{A_{Einstein}}{4G} = 2\pi\sqrt{n_1 n_5 n_p}$$

parameters $(\underset{\times}{R}, \underset{\times}{V}, \underset{\times}{g}, \underset{\times}{\alpha'}, n_1, n_5, n_p)$

horizon is at $r=0$. Singularity?



$L_T = 2\pi R n_1$

n_1 $\ln(256)$ n_1, n_p

NS1, P

NS1, NS5, P II A

↳ why these two? Duality. $\Rightarrow$ I doesn't matte

$M_{4,1} \times T^4 \times S^1$

↖ NS1

T_{z_1} $\overset{z_1 z_2 z_3 z_4}{}$ NS1 − NS5 − P II B

$\downarrow S$

D1 − D5 − P Vafa

permute

~~T_{z_2}/T_{z_4}~~ $T_{z_1 z_4}$: D1 ↔ D5

$AdS_3 \times S^3 \times T_4$ → D1 − D5 $\xrightarrow{S}$ NS1 − NS5 $\xrightarrow[T_4(S^1)]{}$ P−NS5

II B II B II A

→ T_{z_1} P − NS5 $\xrightarrow[S]{}$ P-D5 $\xrightarrow[T_{z_1 \cdots z_4}]{}$ P−D1

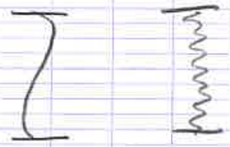
II B

→ S P−NS1

↑

string carrying momentum.

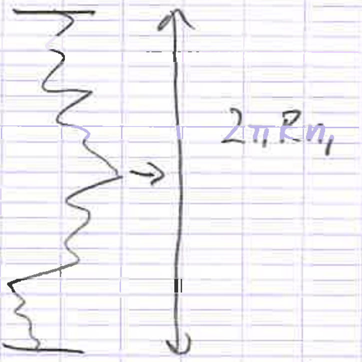
Bound State: (or else many black holes). Bind momentum modes to string ⇒ string carrying traveling wave



How can we partition momentum on a string $P = \frac{2\pi n_p}{L}$

$$\Rightarrow \quad P = \frac{2\pi n_1 n_p}{L_T}$$

total length. $(2\pi R n_1)$



$\frac{8}{2} = 4$ bosons

⇑

(8 fermionic)

8-transverse directions.

$p = \frac{2\pi K}{L_T}$ momentum carried by 1 quantum of K^{th} harmonic

n_i of the harmonic K

$$\boxed{\sum_K n_K K = n_1 n_p}$$

Partitions of integers

$$\# \text{ partitions} \sim e^{2\pi\sqrt{\frac{n_1 n_p}{6}}}$$

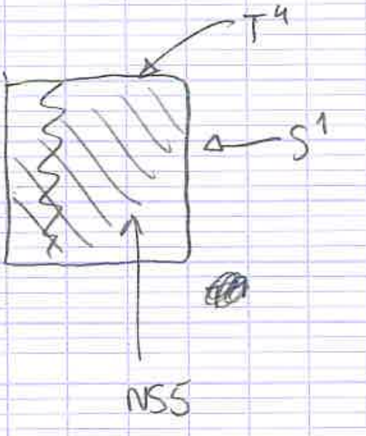
$n_1 n_p$ large

$\Rightarrow \quad \frac{n_1 n_p}{12}$ units of vibration

$$\left(e^{2\pi\sqrt{\frac{n_1 n_p}{12 \cdot 6}}} \right)^{12}$$

$$e^{2\pi\sqrt{2}\sqrt{n_1 n_p}} \quad \Rightarrow \quad S = 2\pi\sqrt{2}\sqrt{n_1 n_p} \quad \checkmark$$

Put 3rd charge back



bind NS1-NS5

NS1 can only vibrate along NS5

$\Rightarrow$ 4 vibrations

4 ferm

$\Rightarrow$ 4+2 = 6 bosons

$$e^{2\pi\sqrt{n_1 n_p}} \to S = 2\pi\sqrt{n_1 n_p} \quad (n_5 = 1)$$

$\Rightarrow$ by duality $S = 2\pi\sqrt{n_1 n_p n_5}$

$\Rightarrow$ $S_{BeK} = S_{micro}$ Strominger & Vafa.

Dabholkar $\to$ R^2 corrections $\frac{A_H}{4G_N} = S_{micro}$ for 2 charges.

2 charges:

$$ds^2_{string} = H[-du\,dv + K dv^2] + \sum_{i=1}^{4} dx^i dx^i + \sum_{i=1}^{1} dz_a dz_a$$

$$H_1 = H_1^{-1} = 1 + \frac{Q_1}{r^2} \quad \text{(notation change)}$$

NS-P1

$$K = \frac{Q_P}{r^2}$$

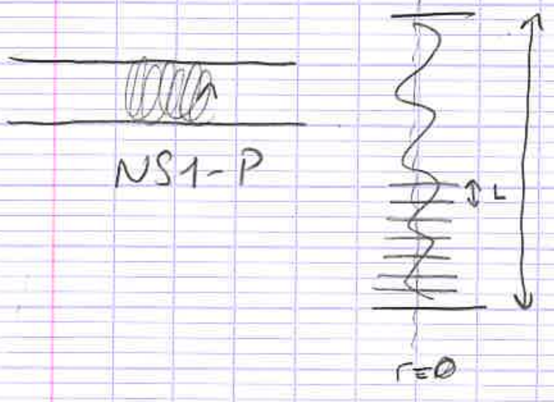
$u = t - y$

$v = t + y$

Unique metric but lots of state. NO state of NS1-P system produces this metric.

String carrying vibrations:

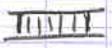
String has no longitudinal vibration modes



$L_T = 2\pi R n_1 = n_1 L$

$t - y \equiv v$

$F_i(v) \quad i=1,\dots,8$

 different strands grow around $r=0$.

$\Rightarrow$ as states get excited it grows.

Metric for a single strand:

~~$ds^2_{string} = H[-dt^2 + dy^2]$~~

$$ds^2_{string} = H[-du\,dv + K dv^2 + 2A_i dx^i dv] + \sum_{i=1}^{4} dx^i dx^i + \sum_a dz_a dz_a$$

$$H^{-1} = 1 + \frac{Q_1}{|\vec{x} - F(\vec{v})|^2} \qquad K = \frac{Q_1 |\dot{\vec{F}}|}{|\vec{x} - F(\vec{v})|^2}$$

$$A_i = \frac{-Q_1 \dot{F}_i}{|\vec{x} - \vec{F}(\vec{v})|^2} \qquad \dot{} = \frac{d}{dv}$$

Many strands

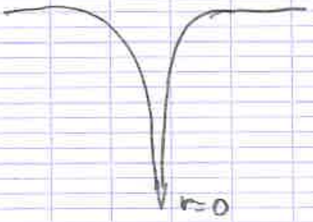
$$H^{-1} \longrightarrow 1 + \sum_{(K)} \frac{Q_K}{|\vec{x} - F_i^{(K)}|^2}$$

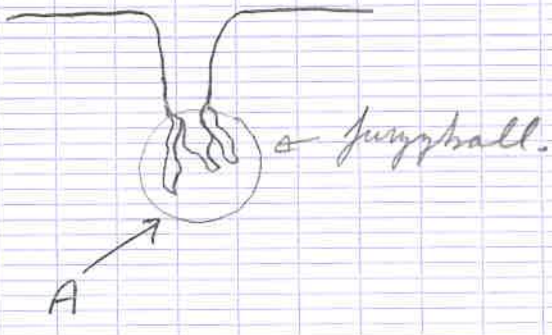
At the end of the day:

$$H^{-1} \rightarrow 1 + \frac{Q_1}{L_T} \int_0^{L_T} \frac{dv}{|\vec{x} - \vec{F}(v)|^2}$$

~~ds^2~~

Naieve metric





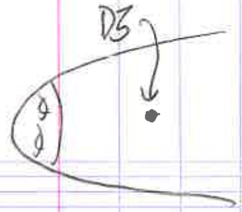
fuzzball states
where $\frac{A}{4G} \sim \sqrt{n_1 n_p}$

no horizon
no ETC
no singularity.

$$\begin{tikzcd} NS1 \arrow[r] \arrow[d] & P \\ D1-D5 \arrow[r] & AdS/CFT \end{tikzcd}$$

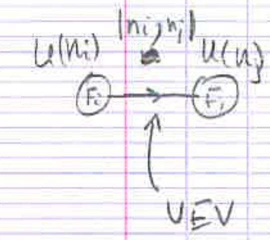
Klebanov

Transparencies available from "Understanding Confinement" Max Planck Institute Workshop.



Given Singularity

$$\Rightarrow \boxed{\text{many collections of } F_i\text{'s}}$$

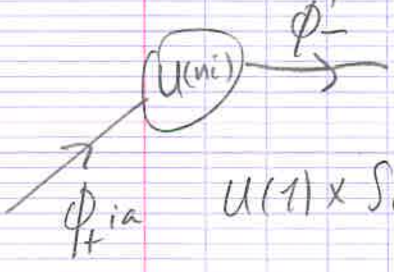


$$ch(F_i) = (\underset{D7}{rK}, \underset{D5}{c_1}, \underset{D3}{ch_2})$$

1. Seiberg duality "mutation"

2. Symmetry breaking
$\Rightarrow$ bound state formation
$ch(F_B) = ch(F_i) + ch(F_j)$

D-term equations



$U(1) \times SU(n_i)$

$$\sum_a |\phi^+_{ai}|^2 - \sum_b |\phi^-_{bi}|^2 = \xi_i$$

U(1) D-term eqn.

FI term

size of
2 cycle.

Seiberg duality:

- Choose node $\overset{F_i}{\vee}$ → apply Seiberg duality to it. Order fractional branes F_j, s.t. lines only go from F_j to F_k if $j>k$ cyclicly.
- Replace $\underset{\substack{j \ll i \\ (\text{or } j>i)}}{ch(F_j)} \longrightarrow ch(F_j) - \underbrace{\underbrace{\chi(F_j, F_i)}_{\substack{\# \text{ of lines from} \\ F_i \to F_j}} ch(F_i)}_{\text{SUM HERE}}$

~~$\Sigma' n_i ch(F_i) = (0,0,1)$~~

$\Sigma\, n_i \underline{\underline{ch(F_i)}} = (0,0,1)$ ← to preserve this change multiplicity

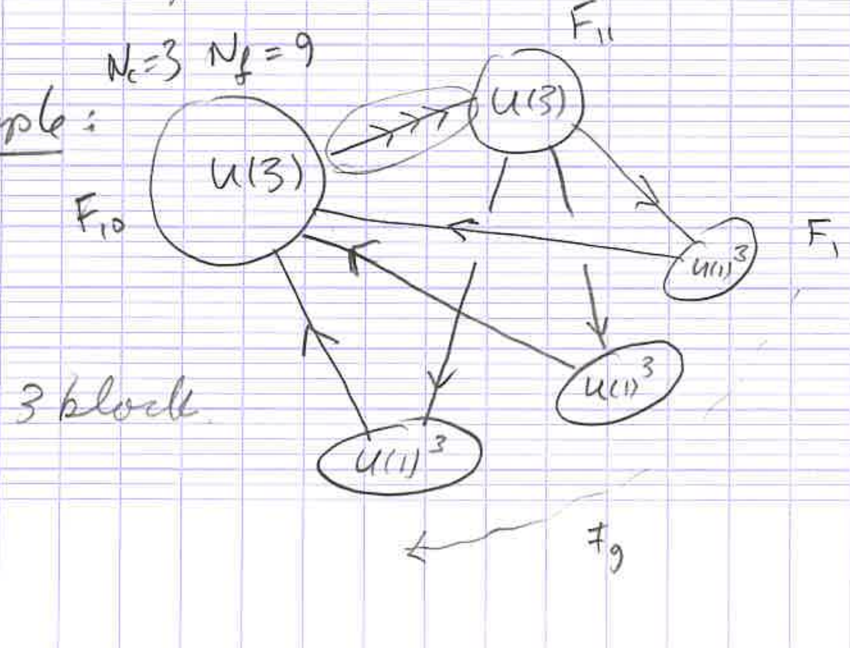
$$n_i \to n_i - N_i = \tilde{n}_i$$

$N_i = \sum_{j<i} \chi(F_i, F_j)\, n_j$ = number of flavors at i^{th} node

$N_c \to N_f - N_c$

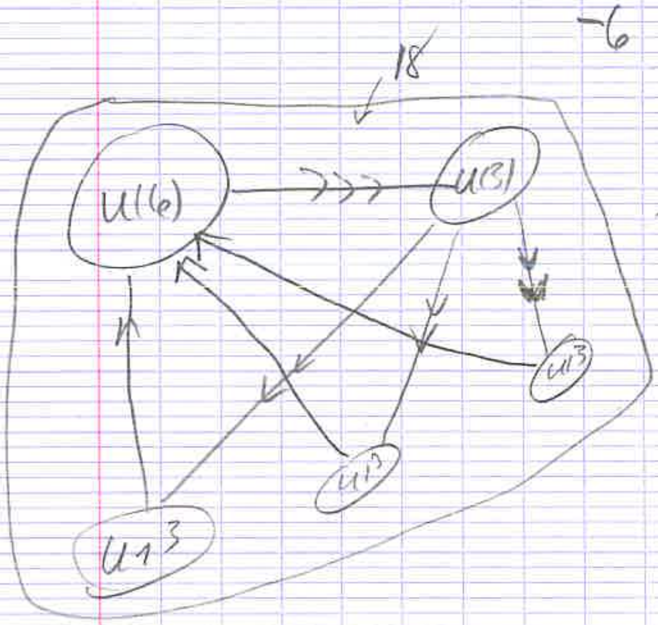
$N_c = 3$ $N_f = 9$

Example:



~~$ch(\tilde{F}_{11}) \rightarrow$~~ $ch(\tilde{F}_{11}) = ch(F_{11}) - 3\,ch(F_{10})$

$$\sum_{i=1}^{9} ch(F_i) + \underset{\substack{\downarrow \\ -6}}{3ch(F_{10})} - 3ch(\underset{\substack{\downarrow \\ \tilde{F}_{11}}}{F_{11}}) = (0,0,1)$$



Seiberg dualized

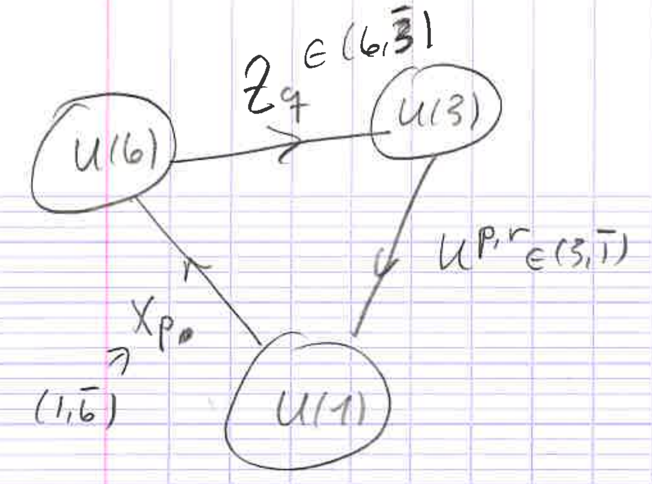
$$\boxed{\sum_j \chi(F_i, F_j)\, n_j = 0}$$ IN = OUT

$$\chi(F_{11}, \underset{\substack{\| \\ (0,0,1)}}{D3}) = 0$$

Lets try to make SM

anomalous U(1)'s cancellation - GS cancellation

~~U(6)~~



$p = 1,\dots,9$

$r = 1,2$

$q = 1,2,3$

$$W = \sum_{p,q,r} c_{pqr}\, X^p Z^q U^{p,r}$$

D-term eqns:

U(1)'s
$$\begin{cases} \sum_r' |U^{p,r}|^2 - |X^p|^2 = \xi_p & p = 1,\dots,9 \\ \sum_p |X^p|^2 - \sum_q |Z^q|^2 = \xi_{10} \\ \sum_q |Z^q|^2 - \sum_{p,r} |U^{p,r}|^2 = \xi_{11} \end{cases}$$

$$SU(6) \quad \sum_p \bar{X}^p T^a X^p = \sum_q Z^q T^a Z^a \qquad T^a \in SU(6)$$

$t_b \in SU(3)$

$$SU(3) \quad \sum_q Z^a t^b Z^b = \sum_{p,r} \bar{u}^{p,r} t^b u^{p,r}$$

→ SU(3) × SU(2)

$s = 1,2,3$
$$X^s = \begin{pmatrix} \phi_1 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \end{pmatrix}, \quad X^{s+3} = \begin{pmatrix} 0 \\ \phi_2 \\ 0 \\ \hline 0 \\ 0 \\ 0 \end{pmatrix} \quad X^{s+6} = \begin{pmatrix} 0 \\ 0 \\ \phi_3 \\ \hline 0 \\ 0 \\ 0 \end{pmatrix}$$

$$Z^1 = \left[\begin{pmatrix} \psi^1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \end{pmatrix} \cdots \begin{pmatrix} 0 \end{pmatrix} \right] \quad Z^2 = \begin{bmatrix} 0 & & 0 \\ \psi_2 & & \\ 0 & \ddots & \\ & & 0 \end{bmatrix} \quad Z^3 = \begin{bmatrix} 0 & & 0 \\ 0 & & \\ \psi^3 & \ddots & \\ 0 & & 0 \end{bmatrix}$$

$SU(6)$ D-term eq. $\Rightarrow \phi_i = \psi_i$

Extra $U(1)$ will be dealt with...

$U^{S,r} = (\chi^r, 0, 0)$

$U^{S+3,r} = \sum_{5}^{7} U^{S+6r} = (0,0,0)$

$U(3) \to U(2)$

$U(6) \to U(3)$

Back to geometry:

Proposal: three new bound state

$$U(3) \times U(2) \times U(1)^3$$

$$\boxed{\begin{aligned} ch(F_o) &= \sum_{i=1}^{3} ch(F_i) - ch(F_{10}) - ch(F_{11}) \\ ch(F_u) &= \sum_{i=4}^{6} ch(F_i) - \underset{\sim}{ch(F_{10})} \\ ch(F_d) &= \sum_{i=7}^{9} ch(F_i) - \underset{\sim}{ch(F_{10})} \end{aligned}}$$

1 F_{11} in boundstate since $3 \to 2$

3 F_{10}'s.

$ch(F_{10}) \to n_{10} = -3$

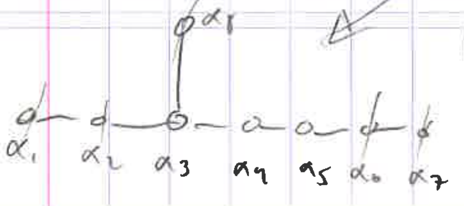
$ch(F_{11}) \to n_{11} = -2$

$\alpha_i = E_{i+1} - E_i \quad i = 1, \ldots, 7$

$\alpha_8 = H - \sum_{1}^{3} E_i$

$K = -3H + \sum_{1}^{8} E_i$

5 - 2cycles killed off.



Dynkin. Diagram (E_8)

$\alpha_4 = E_5 - E_4$

$\varepsilon_4 = K + E_4$

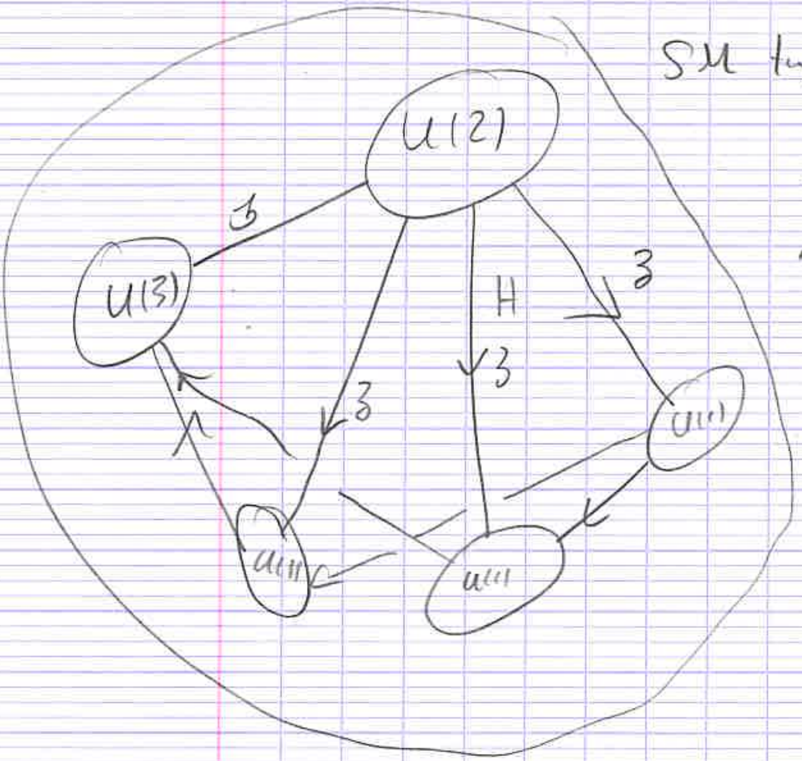
$\lambda_3 = 5K + \sum_{i=1}^{8} E_i$

K

rk	deg	λ_3	ε_4	α_4	D3

table in notes

$\chi(F_i, F_j) = \Big($ 3's appear. $\Big)$



SM type quiver

- 3 higgs
- extra U(1)'s

Mathur

AdS/CFT

Flat Space

Einstein

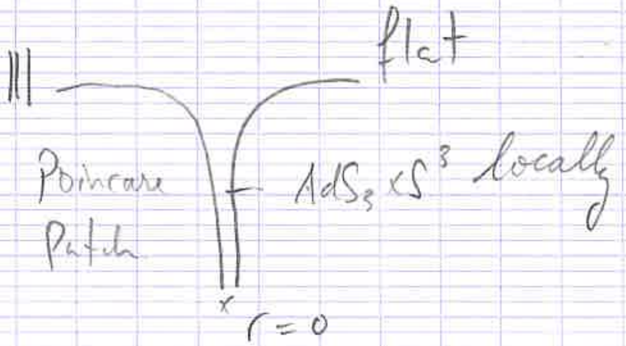
Maldacena: Gravity $\xleftrightarrow{\text{dual}}$ Matter

either but not both.

D3 $AdS_5 \times S^5$

D1-D5 system $AdS_3 \times S^3 \times T^4$

$e^{2\pi\sqrt{2}\sqrt{n_1 n_5}}$



NS1 - P

$$ds^2_{\text{string}} = H\left[-du\,dv - K dv^2 + A_i dx^i dv\right] + \sum_1^4 dx^i dx^i + \sum_a dz_a dz_a$$

$$t_{T^4}$$

$$u = t + y$$

$$v = t - y \leftarrow S^1$$

$$H^{-1} = 1 + \frac{Q_1}{L_T}\int_0^{L_T}\frac{dv}{|\vec{x} - \vec{F}(\vec{v})|^2}$$

$$K = \frac{Q_1}{L_T}\int_0^{L_T}\frac{dv\,|\vec{\dot{F}}|^2}{|\vec{x} - \vec{F}(\vec{v})|^2}$$

$$A_i = \frac{-Q_1}{L_T}\int_0^{L_T}\frac{dv\,\dot{F}_i(v)}{|\vec{x} - \vec{F}(\vec{v})|^2}$$

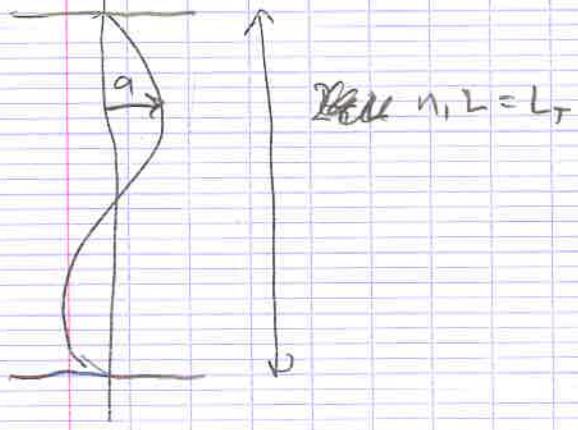
$$\begin{tikzcd}
Q_1 & Q_P & \\
\mathrm{NS1-} \arrow[u] \arrow[d] & \mathrm{P} \arrow[u] \arrow[d] \arrow[r, mapsto] & \mathrm{D1-D5} \\
\mathrm{D5} \arrow[d] & \mathrm{D1} \arrow[d] & \\
Q_5 & Q_1 &
\end{tikzcd}$$

D1-D5 $$ds^2_{\text{string}} = \sqrt{\frac{H}{1+K}}\left(-(dt^2 - A_i dx^i)^2 + (dy + B_i dx^i)^2\right)$$

$$+ \sqrt{\frac{\overset{1}{\cancel{H}} - K}{H}}\,dx^i dx^i + \sqrt{H(1+K)}\,dz_a dz_a$$

$dB = * dA$

Special Case:



$F_1 = a \cos \omega v$

$F_2 = a \sin \omega v$

$F_3 = F_4 = 0$

$\omega = \frac{1}{n_1 R}$

1 turn of uniform helix

$$H^{-1} = 1 + \frac{Q_1}{2\pi} \int_0^{2\pi} \frac{ds}{|x^1 - a\cos\xi|^2 + (x^2 - a\sin\xi)^2 + x_3^2 + x_4^2}$$

$X_i \to$ polar coords.

$$H^{-1} = 1 + \frac{Q_1}{r^2 + a^2 \cos^2\theta}$$

"Near region"

$$ds^2 = -(r^2+a^2)\frac{dt^2}{\sqrt{Q_1 Q_5}} + \frac{r^2 dy^2}{\sqrt{Q_1 Q_5}}$$

$$+ \sqrt{Q_1 Q_5}\frac{dr^2}{r^2+a^2}$$

$$+ \sqrt{Q_1 Q_5}\left[d\theta^2 + \cos^2\theta\left(d\psi - \frac{a\,dy}{\sqrt{Q_1 Q_5}}\right)^2\right.$$

$$+ \sqrt{\frac{Q_1}{Q_5}}\, dz_a dz_a + \sin^2\theta\left(d\phi - \frac{a\,dt}{\sqrt{Q_1 Q_5}}\right)^2$$

$$\psi' = \psi - \frac{a}{\sqrt{Q_1 Q_5}} y$$

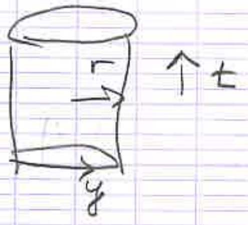
$$\phi' = \phi + \frac{a t}{\sqrt{Q_1 Q_5}}$$

$$r' = \frac{r}{a}$$

$$ds^2 = \sqrt{Q_1 Q_5}\left\{-(1+r'^2)\frac{dt^2}{R^2} + r'^2\frac{dy^2}{R^2} + \frac{dr'^2}{1+r'^2}\right. \leftarrow \text{Global } AdS_3$$

$$+\sqrt{Q_1Q_5}\left[d\theta^2 + \cos^2\theta\, d\psi'^2 + \sin^2\theta\, d\phi^2\right]$$

$\uparrow S^3$



$p = \frac{2\pi}{L}$

$$P = \frac{2\pi n_p}{L} = \frac{2\pi n_1 n_p}{L_T}$$

n_1

n_p

Momentum comes in units of $\frac{2\pi}{L_T}$, not $\frac{2\pi}{L}$

$\quad\quad\quad\quad\quad\quad\quad\quad \shortparallel$

$\quad\quad\quad\quad\quad\quad\quad \frac{2\pi}{n_1 L}$

When you make a <u>bound</u> state of n_1 strings $\Rightarrow$ p excitations come in fractional units $\frac{1}{n_1}$

$p \sim \frac{2\pi}{L}$

elementary string

$\boxed{2\pi\sqrt{n_1 n_p}}$

P1 D5

effective string

$n_1 L \mapsto n_1 n_5 L$

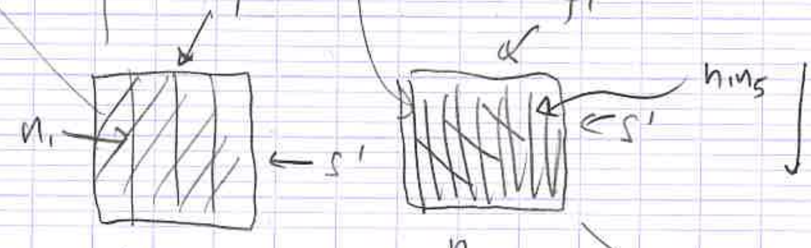
D1 D5

$$\begin{tikzcd} NS1 - P \arrow[r] & D5 - D1 \end{tikzcd}$$

$n_1 \quad n_p \quad\quad n_5 \quad n_1$

$n_1 n_p$ fractional units of momentum

$\to n_1 n_5$ fractional branes $\left(\frac{T_{D1}}{D5}\right)$

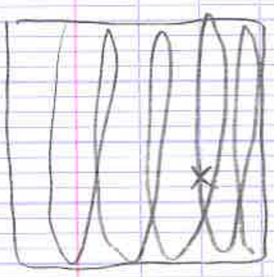


$n_5 = 1$

n_5

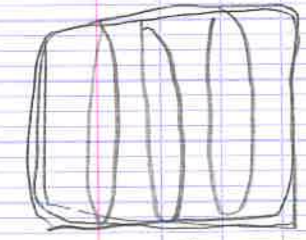
$2\pi\sqrt{n_1 n_5 n_p}$

CFT for D1D5 $n_1 n_5$ strands of a string wrapped along S^1

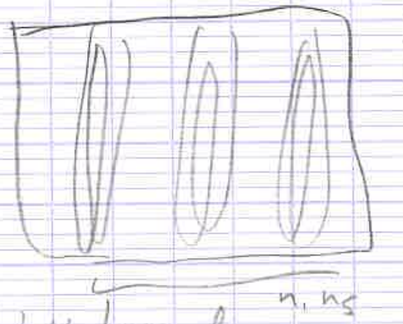


T^4 will not vibrate (?)

D1-D5



glued to D5, so still bound



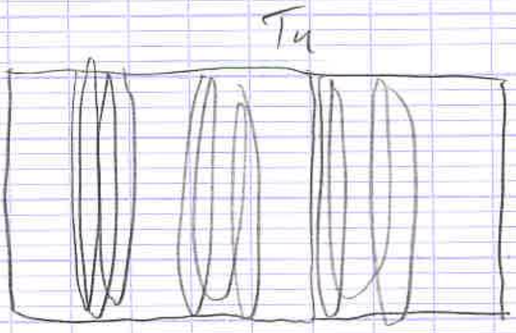
$P(n_1 n_5) = e^{2\pi \sqrt{n_1 n_5} \sqrt{2}}$

Knot software

Typing

$\alpha_{-k_1} \alpha_{-k_2} \cdots \alpha_{-k_n} |0\rangle$

T_4

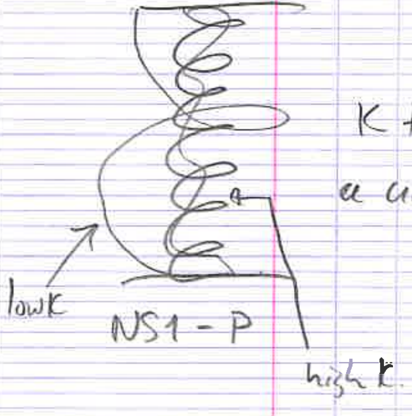


S^1

geometry? →

$\sum_i k_i = n_1 n_5$

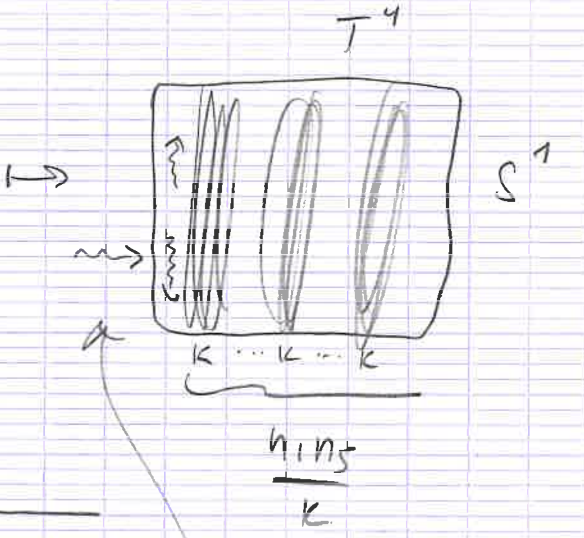
Special Subclass of states

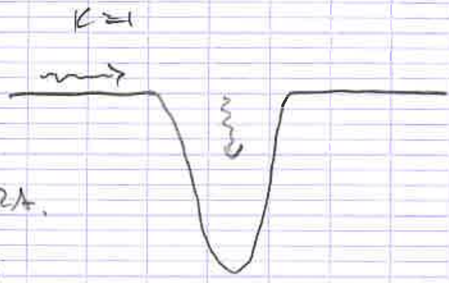


K turns of a uniform helix

$$(\alpha_{-K})^{\frac{n_1 n_p}{K}}|0\rangle$$

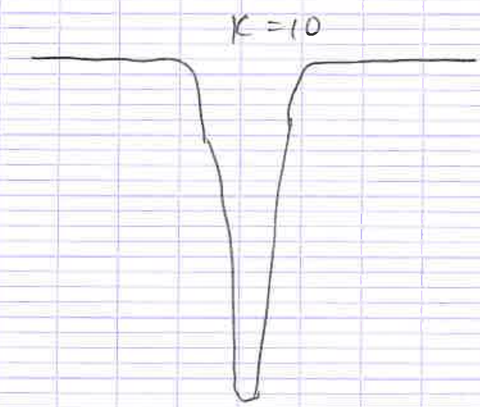
$\longmapsto$





P^{abs}_{SUGRA}

σ^{abs}_{SUGRA}

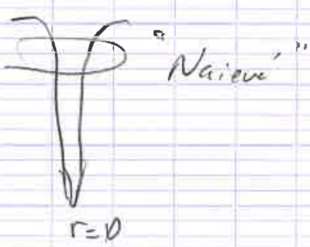


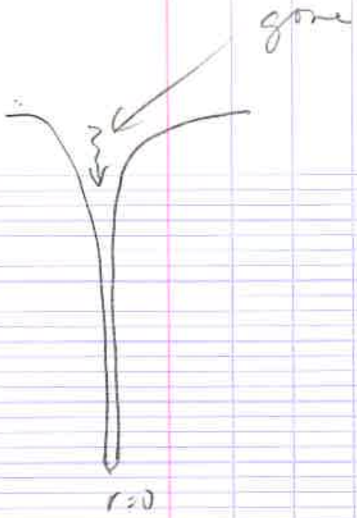
Flat Space

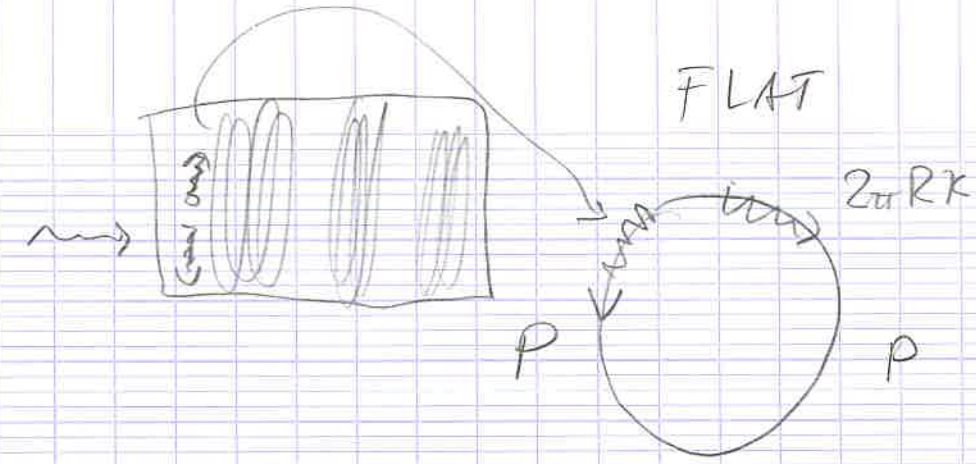
P^{abs}_{CFT}

σ^{abs}_{CFT}

$P_{CFT} = P_{SUGRA}$ '90's.

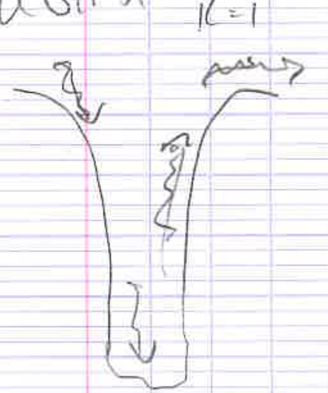






$$\Delta t = \frac{2\pi RK}{2} = \pi RK$$

SUGRA

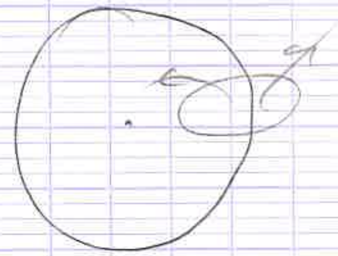


Backreaction issue

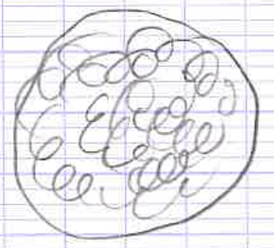
$$\Delta t_{EFT} = \Delta t_{SUGRA}$$

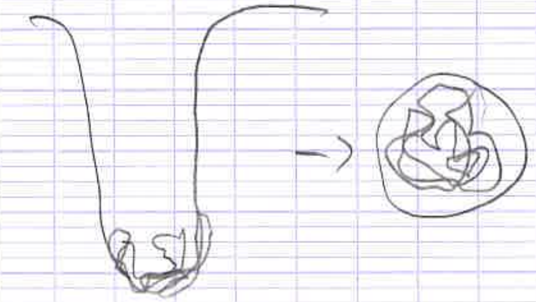
1. Hawking paradox severe

$\lesssim l_p, l_s$
VAC UNIQUE } ⇒ info loss



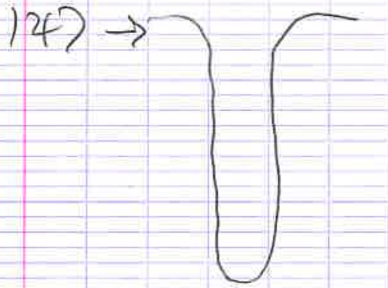
2. 2 chg system $S = 2\pi\sqrt{2}\sqrt{n_1 n_5}$



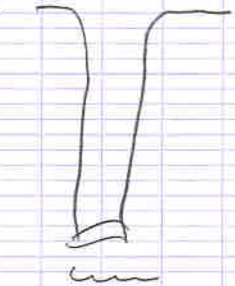


$$\frac{A}{4G} \sim \sqrt{n_1 n_5}$$

3 charge: Some solutions



Naive solution



Find all states for ~~D5~~ D1D5 3chg

$\Delta t_{CFT} = \Delta t_{SUGRA}$

3charge

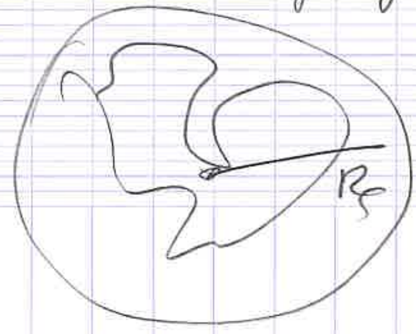
$\Delta t_{CFT} \approx \Delta t_{SUGRA}$

Estimate size of D1-D5-P → ~ R_s

Fractionation:

$T \sim \frac{1}{n_1 n_5 n_p}$

low tension strings go FAR



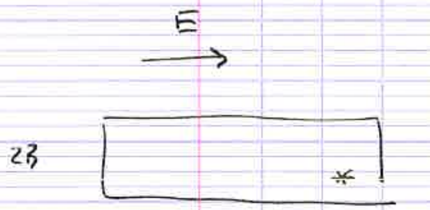
6|20|04

Zaldarriegia I

(Black hole gas)

Mellisinos.

Download good asstronomical the data & pictures.



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phys 1140 lab

6/20/04

51

Solitons:

Plan:
1) Instanton
2) Monopoles
3) Vortices
4) Domain Walls (Kinks)

§1 Instantons:

soln's $F=*F$ '75
Donaldson 4-Manifolds '80
SW soln - Nekrasov '2003

Pure $su(N)$ YM $\quad S=\int d^4x \frac{1}{4e^2}\,\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$

$$= \int d^4x\, \frac{1}{4e^2}\,\mathrm{Tr}\left(F_{\mu\nu}-*F_{\mu\nu}\right)^2 + \underbrace{\frac{1}{4e^2}\mathrm{Tr}\, F_{\mu\nu}\tilde{F}^{\mu\nu}}_{\text{total der.}}$$

$$\Rightarrow S \geq \int d^4x\, \frac{1}{4e^2}\,\partial_\mu\left(A_\nu F_{\rho\sigma}+\frac{2}{3}A_\nu A_\rho A_\sigma\right]\underset{\epsilon_{\mu\nu\rho\sigma}}{\uparrow}$$

as $x\to\infty$ $\quad A_\mu \to i g\,\partial_\mu g^+ \quad g\in SO(N)$

$\Rightarrow \exists$ map $S^3_\infty \mapsto SU(N) \quad \pi_3(SO(N))\cong \mathbb{Z}$

$S \geq \frac{4\pi^2}{e^2}k \quad k\in\mathbb{Z}$

with equality iff $F_{\mu\nu}=*F_{\mu\nu}$ soln's of this 1st order eqn satisfies full 2nd order eqns.

Ex: $k=1$ $su(2)$: $\quad A_\mu = \dfrac{\rho^2(x-X)_\mu}{(x-X)^2\left((x-X)^2+\rho^2\right)} \qquad \bar{\eta}^a_{\mu\nu} = g\,\sigma^a g^+$

pauli

$g\in su(2)$

8 param X

't Hooft matrices: $\bar\eta^1=\begin{pmatrix} &&&-1\\ &&1&\\ &-1&&\\ 1&&& \end{pmatrix} \quad \bar\eta^2=\begin{pmatrix} &&-1&\\ &&&-1\\ 1&&&\\ &1&& \end{pmatrix} \quad \bar\eta^3=\begin{pmatrix} &1&&\\ -1&&&\\ &&&-1\\ &&1& \end{pmatrix}$

like AdS/CFT ← $\left[\begin{array}{l}\text{local g.T.} \to \text{trivial} \\ \text{global g.T} \to \text{nontrivial}\end{array}\right]$

Sol'n has 8 parameters (or collective coordinates)

- 4 translations X_μ
- 1 scale size ρ
- 3 SU(3) orientation modes g.

What about $K=1$ instanton in $SU(N)$? $A_\mu = \left(\begin{array}{c|c} A_\mu^{SU(2)} & 0 \\ \hline 0 & 0 \end{array}\right)$

Act with $SU(N)/S[U(N-2)\times U(2)]$

$\Rightarrow (N^2-1) - ((N-2)^2 + 4 - 1) = 4N - 8$

$\Rightarrow$ $k=1$ instanton has $4N$ collective coordinates

Atiyah - Singer index theorem:

$\Rightarrow$ Charge K instanton in $SU(N)$ gauge group has $\boxed{4KN}$ parameters

↑ not well understood for <u>not</u> widely separated

<u>The Moduli space:</u>

Space of solutions to $*F = F$ with fixed charge K & gauge group $SU(N)$

$\boxed{\mathcal{L}_{K,N}}$

$$\boxed{\dim \mathcal{L}_{K,N} = 4KN}$$

The metric on $\mathcal{L}_{K,N}$:

Take a solution $A_\mu \rightarrow A_\mu + \delta A_\mu$

$$F = *F \Rightarrow \mathcal{D}_\mu \delta A_\nu = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\mathcal{D}_\rho \delta A_\sigma \quad \text{—} \; (\overset{\text{self}}{S}\overset{\text{dual}}{D})$$

$$\mathcal{D}_\mu X = \partial_\mu X - i[A_\mu, X]$$

(Black holes?)

choose gf condition $\mathcal{D}_\mu \delta A_\mu = 0$ — $(\overset{\text{gauge}}{G}\,\overset{\text{fixing}}{F})$

For every collective coordinate X^α $\alpha = 1, \ldots, 4KN$

We can take $A_\mu(x_\nu, X^\alpha)$ and generate

$$\delta_\alpha A_\mu = \frac{\partial A_\mu}{\partial X_\alpha} + \underbrace{D_\mu \Omega_\alpha} \quad \text{(Zero modes).}$$

↑ chosen to satisfy GF

tangent vectors on config space.

Metric on $\mathcal{L}_{K,N}$:

$$\boxed{g_{\alpha\beta} = \int d^4x \, \frac{1}{e^2} \, \mathrm{Tr}\left[\delta_\alpha A_\mu, \delta_\beta A_\nu\right]}$$

$A_\mu = \tau^a A_\mu^a$

$\mathrm{Tr}[\tau^a, \tau^b] = \underline{\mathrm{Tr}(f^{abc}\tau_c)}$

Properties of $g_{\alpha\beta}$:

- smooth except for localized singularities at $\rho = 0$
- hyper Kahler.
- encodes information about solitons

Aside:

Kahler:

$J^\alpha{}_\beta$

$z^\alpha = x^\alpha + iJ^\alpha{}_\beta x^\beta$

Hyper Kahler:

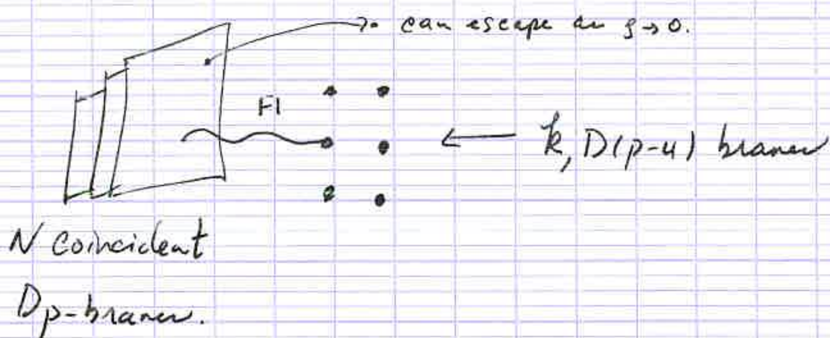
3 J's $\quad J^{(i)} J^{(j)} = -\delta^{ij} + \varepsilon^{ijk} J_k$

ADHM construction: '79 (Twistor Space)

Witten, Douglas '95 D-branes

ADHM construction:

Type II



Dp-branes

$d=p+1$ U(N) SYM

+ coupling to RR fields

$$\mathrm{Tr}\int_{D_p} d^{p+1}\, C_{(p-3)} \wedge F \wedge F$$

⇒ Instanton in Dp-branes ⇒ $\frac{4\pi^2}{e^2}\int d x^{p-3}\, C_{(p-3)}$

$$\boxed{\text{Instanton in Dp-brane IS a D(p-4) brane}}$$

What is the theory on the D(p-4) branes?

$\underline{p=3|}$ U(k) gauge theory (SYM) in $d=0+0$ dimensions

10 adjoint scalars (X^μ, X^n) $\mu=1,2,3,4$
$n=5,\dots,10$

Dp-D(p-4) strings ⇒ hypermultiplets $\psi^\alpha_a, \tilde\psi^\beta_b$

$\alpha=1,\dots,K$

$a=1,\dots,N$

ψ in $(k, \bar{N})$ of ~~SU(N)~~ $U(K) \times SU(N)$

$\tilde{\psi}$ in $(\bar{k}, N)$

8 supersymmetries ⟹ unique potential that must be satisfied

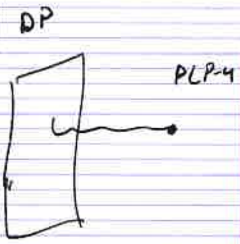
$$V = \frac{1}{g^2} \sum_{m,n=1}^{5} [X_m, X_n]^2 + \sum_{n=5}^{10}{}' \sum_{\mu=1}^{4} [X_m, X_\mu]^2$$

$$+ \sum_{a=1}^{N} \left(\psi^{a\dagger} X_n^2 \psi_a + \tilde{\psi}_a X_n^{*} \tilde{\psi}^{aT} \right)$$

tensor product of gauge indices

U(K) valued.

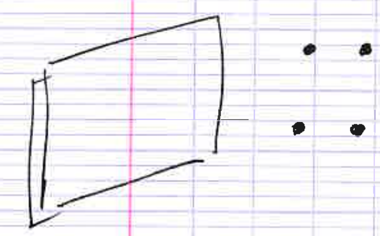
$$+ g^2 T_r \left(\sum_{a=1}^{N}{}' \psi_a \psi_a^\dagger - \psi_a^\dagger \tilde{\psi}^a + [z, z^\dagger] + [w, w^\dagger] \right)^2 \quad \text{D term}$$



$$+ g^2 T_r \left| \sum_a{}' \psi_a \tilde{\psi}_a^\dagger + [z, w] \right|^2 \quad \text{F-term.}$$

Sol'n's to $V = 0$:

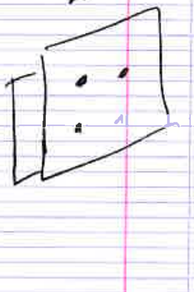
1. $\psi = \tilde{\psi} = 0$ with all other fields diagonal. (Coulomb) branch



2. $X_m = 0$, with $\sum_a \psi_a \psi_a^\dagger - \tilde{\psi}_a^\dagger \tilde{\psi}_a + [z, z^\dagger] + [w, w^\dagger] = 0$

Higgs branch

$$\sum_a \psi_a \tilde{\psi}_a + [z, w] = 0$$



$$\dim(M_{Higgs}) = 2KN + 2KN + 4K^2 - K^2 - 2K^2 - \underbrace{K^2}_{\text{gauge trans. } U(K)} = 4KN$$

$$= \dim(\mathcal{L}_{K,N})$$

Summary:

Gauge Thy w/ 8 supercharges ($N=2$, $D=4$), adjoint hyper multiplet Z, w, + N fundamental hypermultiplet

$\mathcal{M}_{HIGGS} = \mathcal{L}_{KIN}$

cf AdS/CFT

6/20/05

G. Kane

S. Martin hep-ph/9709356
Binetruy - Book
G Kane latest article.

Murayama's Notes:

http://hitoshi.berkley.edu. TASI 05

The offshell onshell question

SUSY - Auxiliary fields
String Thy - offshell does not seem to exist

~~SUSY~~ and ~~Lorentz~~ by VEVS' of ~~a~~ nontrivial reps of Lorentz group. What about lots of VEV's of ~~a~~ vectors $V^\mu_{(i)}$ with random orientations? i.e.

Large N $\cancel{\sum_i V^\mu_{(i)}} =$

$$\sum_{i=1}^{N_1} V^\mu_{(i)} \approx 0 \Rightarrow \text{NO } \text{LOR}\cancel{\text{ENTZ}}$$

Symmetry Breaking from "within" i.e. w/o adding more DOF's (like Higgs)

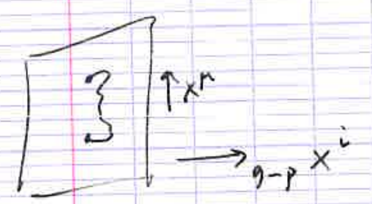
Model Building With D-branes:

I. II A intersecting brane models

II. II B magnetized D-brane, branes at singus.

III. D-branes & fluxes & compacts.

Dp-branes



N Dp-branes

U(N) gauge sym.
9-p real scalars in adjoint
+ fermions

} V. m
(p+1) dim
16 SUSY's.

p even II A $\qquad$ D3 → $N=4$ $U(N)$ SYM
p odd II B

$Q_{L,R}$ $\qquad$ $Q = \epsilon_L \cdot Q_L + \epsilon_R \cdot Q_R$

$$\epsilon_L = \Gamma^0 \cdots \Gamma^p \epsilon_r$$

Worldvolume dynamics:

$$S = \int d^{p+1}x \, e^{-\phi} \sqrt{\det(P[G] + F)}$$

$$+ \int_{D_p} C_{p+1} + \int C_{p-1} \wedge \mathrm{tr} F + \frac{1}{2} \int C_{p-3} \wedge \mathrm{tr} F^2$$

→ tension + SYM + corrections + charges.

How to obtain chirality:

$$SO(10) \to SO(6) \times SO(4)$$

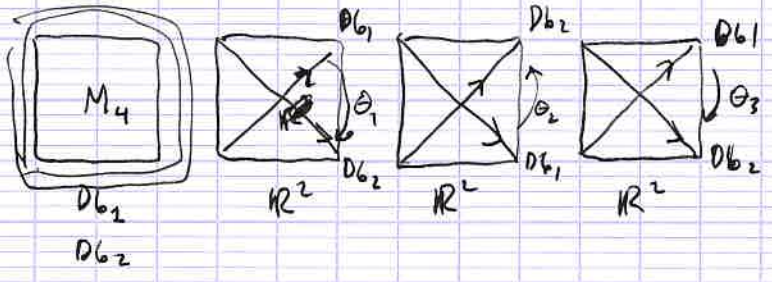
$$\underline{16} \to (4,2) + (\bar{4},2')$$

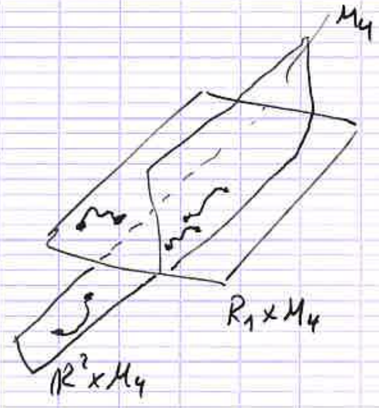
Distinguish between 2 & 2' ⇒ must distinguish 4 & $\bar{4}$.

⇒ the config. must have orientation in 6D.

IIA intersecting D6 branes

$$M_{10} = M_4 \times \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2$$





$6_1 6_1$ $U(N_1)$ SYM on $M^4 \times \mathbb{R}^2$ (superpartners)

$6_2 6_2$ $U(N_2)$ SYM on (adj: stuff). fundamental

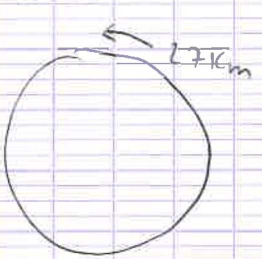
$6_1 6_2$ + $6_2 6_1$ 4d Chiral fermion $(\square_1, \bar{\square}_2)$ + light scalars.

$$\alpha' m^2 = (\theta_1 + \theta_2 + \theta_3)$$

J. Hewitt

Collider Physics

LHC



Magnets

Webpage

Unitarity in Mandelstam variables.

Colliders:

Hadron:

			$\sqrt{s}$ TeV	$\mathcal{L}(cm^{-2}s^{-1})$
Tevatron	$(p\bar{p})$	(Fermilab)	1.98	2.1×10^{32}
HERA	(ep)	(Germany)	0.314	1.4×10^{31}
LHC	(pp)		14	10^{34}
VLHC				

$\int\mathcal{L}dt$	Length	Dates
RUN I 100 pb^{-1} RUN II 500 pb^{-1}	6	mid '80's '02-'09 mid '90's
~400 pb^{-1}	6	1994-2006
Low 10 fb^{-1} High 10 fb^{-1}	27	2008 2012

upgrade.

e^+e^-:

SLC	(80% polarized beam e^-)	m_Z	2.5×10^{30}	500,000 Z's	3	'89-'96
LEP I		m_Z	2.4×10^{31}	4 million Z's		'89-'95
LEP II		130-205 GeV	10^{32}	250 pb		96-2000
ILC	(80% e^- 60% e^+)	.5 - 1 TeV	2×10^{34}	200 fb^{-1}		? ? ? ? ? (2018)
CLIC		3 - 5 TeV				

weak mixing angle.

Higgs.

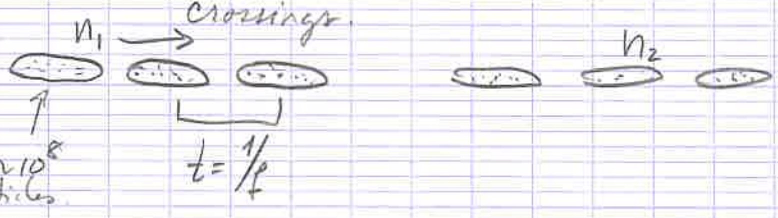
+20yrs.

Laser or plasma accelerators? Better ways to accelerate.

Planing

$\# \text{events} = \sigma \int \mathcal{L}\, dt$ $1 \text{ yr} \equiv 10^7 \text{ sec}$ = Snowmass year

Collider Luminosity $\mathcal{L} = \underset{\text{frequency of bunch crossings}}{f}\ \underbrace{n_1 n_2}_{\text{particles in bunches}} / \underset{\text{transverse profile of beams}}{a}$

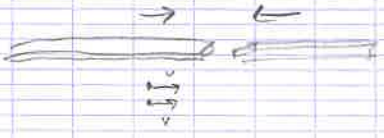


9 interactions per bunch crossing

increasing frequency → LHC $\boxed{1 \text{GHz} = \text{event rate}}$ ← HUGE

small a's → ILC $\boxed{a \sim 100 \text{ nm}^2}$ ← SMALL

↓
"ribbons"



LHC:

3 levels of trigger

- each level reduces by 10^{-2}
- Grid.

e^+e^- Advantages:

1) Well defined initial state.
2) Initial E_{beam} is known and tunable. Study At threshold → precision
3) beam polarization.
4) Clean environment.

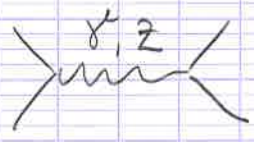
Disadvantages:

1) Large synchrotron radiation $\gamma^{e} \sim \left(\frac{E}{m_e c}\right)^4$

2) $\sigma_{pt} := \sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-) = \frac{4\pi^2\alpha}{3s}$

3) Need to know $\sqrt{s}$.

$\sigma \sim$ EWK strength



R-ratio: $\frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma_{pt}}$

Hadron Colliders:

Advantages:

1) $m_p > m_e$ Fewer synchrotron radiation.

2) Broad energy reach

3) Big cross-sections → strong interactions

total $\sigma(pp) \sim 2\pi R^2 \sim 100$ mb ! @ LHC

4) many possible channels

Disadvantages:

1) Big crosssections, big QCD background
Large background of $b\bar{b}$.

2) No polarization.

3) Unknown quantum state

4) Unknown $\sqrt{s}$ of collision.

Lots of Web Stuff . .

Joanne Hewitt post on the web.

2) Monopoles

$$B_i \sim \frac{g\hat{r}_i}{r^2}$$

· $\vec{\nabla}\cdot\vec{B}=0$

· never been observed

· Dirac: singular gauge pot'ls ok if $eg = 2\pi N$ $N\in\mathbb{Z}$.

· 't Hooft, Polyakov monopole

SU(N) gauge theory:

$$S=\int d^4x\ \mathrm{Tr}\Big[\frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2e^2}(\mathcal{D}_\mu\phi)^2 \qquad \mathcal{D}_\mu\phi=\partial_\mu\phi - i[A_\mu,\phi]$$

$$\langle\phi\rangle = \mathrm{diag}(\phi_1,\dots,\phi_N)=\vec{\phi}\cdot\vec{H} \quad \longleftarrow \text{cartan subalgebra.} \qquad \sum_i\phi_i=0$$

$\phi_a\neq\phi_b$ for $a\neq b \Rightarrow SU(N)\rightarrow U(1)^{N-1}$

This theory has magnetic monopoles

Gauge Equivalent vacua:

$$\pi_2\left(SU(N)/U(1)^{N-1}\right)\simeq \pi_1\left(U(1)^{N-1}\right)\cong\mathbb{Z}^{N-1} \quad \left(\text{N-1 types of Solitons}\right)$$

Goddard & Olive paper.

∃ N-1 types of solitons.

Why magnetic fields?

$$\mathcal{D}_i\phi \xrightarrow[r\to\infty]{} \frac{1}{r^2}$$

but if VEV $\partial_\theta\phi\sim\frac{1}{r} \Rightarrow$ need $A_\theta\neq0$

$\uparrow$ θ coord. on S^2_∞ –

in fact $A_0 \sim \frac{1}{r} \Rightarrow B \sim \frac{1}{r^2}$ at ∞

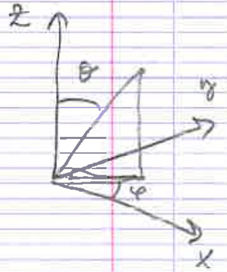
$$B_i = \vec{g}\cdot\vec{H}(\theta)\,\frac{\hat{r}_i}{r^2}$$

Unitary gauge (sleazy gauge)

Pick $\langle\phi\rangle = \phi\cdot\vec{H}$ = constant at ∞.

$$B_i = \mathrm{diag}(g_1,\dots,g_N)\,\frac{\hat{r}_i}{r^2} \qquad \sum_i g_i = 0$$

<u>Dirac quantization</u>: What values of $\vec{g}$ are allowed.



$$A^N = \frac{1}{r}\,\frac{(1-\cos\theta)}{\sin\theta}\,\vec{g}\cdot\vec{H}\,\hat{e}_\varphi \quad \text{good except at } \theta=\pi$$

$$A^S = -\frac{1}{r}\,\frac{(1+\cos\theta)}{\sin\theta}\,\vec{g}\cdot\vec{H}\,\hat{e}_\varphi \quad \text{" " " } \theta=0$$

Dirac String <-> singularity.

At overlap $\exists$ a gauge transf s.t. $A_\mu^N = U(\partial_\mu + A_\mu^S)U^{-1}$

Soln: $U(\theta,\phi) = e^{i\vec{g}\cdot\vec{H}\phi}$

the Gauge transf must be single valued $2\pi\vec{g}\cdot\vec{H} \in \mathbb{Z}\cdot 2\pi$

$$\Rightarrow \vec{g} = (g_1,\dots,g_N) \in \mathbb{Z}^N$$

New notation:

$$\vec{g} = \sum_{a=1}^{N-1} n_a \vec{\alpha}_a$$

$n_a \in \mathbb{Z}$. simple roots of $SU(N)$.

$$\vec{\alpha}_1 = \overbrace{(1,-1,0\cdots,0)}^{N}$$

$$\vec{\alpha}_2 = (0,1,-1,0,\cdots,0)$$

$$\vdots$$

$$\vec{\alpha}_{N-1} = (\qquad\qquad)$$

Monopole Equations: $\partial_0 \equiv 0$, $A_0 \equiv 0$

$$H = T_2 \int d^3x \, \frac{1}{2e^2} B_i^2 + \frac{1}{2e^2}(D_i\phi)^2$$ Bogomolnyi trick

$$= T_2 \int d^3x \, (B_i - D_i\phi)^2 + \frac{1}{e^2} T_2 B_i D_i \phi$$

$$\Rightarrow \mathcal{H} \geq -\frac{1}{e^2}\int d^3x \, T_2\, \partial_i (B_i \phi)$$ using $D_i B_i = 0$.

$$= \frac{2\pi}{e^2} |\vec{g}\cdot\vec{\phi}|$$

with equality iff $\boxed{B_i = D_i\phi}$ monopole

$\boxed{B_i = -D_i\phi}$ anti-monopole.

Note: These are same as $F = *F$

- $\partial_4 \equiv 0$, relabel $A_4 = \phi$

and give $\langle\phi\rangle = \vec{\phi}\cdot\hat{n}$

An example of a solution:

1 monopole in SU(2) $(\vec{g} = \vec{\alpha}_1)$

SU(2) $\alpha = 1,2,3$

~~$\phi = \frac{\hat{r}^\alpha}{r}\left[vr\coth(vr) - 1\right]$~~

$$\phi = \frac{\hat{r}^a \sigma^a}{r}\left[vr\coth(vr) - 1\right] \qquad a = 1, \dots, 3$$

as $r \to \infty$ $\langle\phi\rangle \to v\hat{r}^i\sigma^i$

$$A_\mu = -\varepsilon_{\mu j}\frac{\hat{r}^j\sigma^j}{r}\left(1 - \frac{vr}{\sinh(vr)}\right)$$

$v = -\phi_1 = \phi_2$

BPS Prasad-Sommerfeld BPS states (Witten Olive).

Collective coordinates:

4 collective coordinates :
- 3 translations $\vec{r} \mapsto \vec{r} - \vec{x}$
- Global U(1) G.T.'s

Size of monopole $\sim 1/\langle\phi\rangle \sim 1/M_W$

Moduli Space : (of magnetic monopoles)

The space of sol^ns to monopole eqns $B_i = D_i\phi$ denoted $M_{\vec{g}}$

Erik Weinberg

Callias index theorem.

- $\dim M_{\vec{g}} = 4\sum_{a=1}^{N-1} n_a$

<u>metric on moduli space $\mathcal{M}_{\vec{g}}$:</u> Defined same as last time.

Look for zero modes: $\delta_\alpha A_i \quad \delta_\alpha \phi \equiv \delta_\alpha A_4$

$$\underset{\substack{\uparrow \\ \alpha = \#\text{ of} \\ \text{zero} \\ \text{modes} \\ = \dim \mathcal{M}_{\vec{g}}}}{\delta_\alpha A_i}$$

$$g_{\alpha\beta} = \int d^3x \, \frac{1}{e^2} \mathrm{Tr}\left[\delta_\alpha A_\mu \delta_\beta A_\mu + \delta_\alpha \phi \, \delta_\beta \phi \right]$$

- hyper Kahler
- smooth

<u>Physical Interpretation of metric:</u>

For moving monopoles make ansatz that $\underset{\substack{\uparrow \\ \text{collective coords.}}}{X^\alpha} = X^\alpha(t)$ (adiabatic motion)

$$A_\mu \mapsto A_\mu(X^\alpha(t)) \qquad \text{but need to solve Gauss law}$$
$$\phi \mapsto \phi(X^\alpha(t))$$

$$\mathcal{D}_0 A_i - i[\phi, \mathcal{D}_0 \phi] = 0 \qquad \text{Gauss constraint}$$

$$\delta_\alpha A_\mu = \partial_\alpha A_\mu - \mathcal{D}_\mu \Omega_\alpha$$

and Gauss law is solved if we set $A_0 = \Omega_\alpha \dot{X}^\alpha$

$$\Rightarrow \quad E_i = \delta_\alpha A_i \dot{X}^\alpha$$
$$\mathcal{D}_0 \phi = \delta_\alpha \phi \, \dot{X}^\alpha$$

$$\Rightarrow \quad S = \mathrm{Tr} \int d^4x \, \frac{1}{4e^2} F + \frac{1}{2e^2} (\mathcal{D}\phi)^2 = \int dt \left(M_{\substack{\text{monopole} \\ \text{mass}}} + g_{\alpha\beta} \dot{X}^\alpha \dot{X}^\beta \right)$$

{ Monopole collision in
{ moduli space

Examples:

- 1 monopole in $SU(2)$ $\quad \mathcal{M}_1 \cong \mathbb{R}^3 \times S^1$

- 2 monopoles in $SU(2)$ $\quad \mathcal{M}_2 \cong \dfrac{\mathbb{R}^3 \times S^1 \times \mathcal{M}_{AH}^{\Delta}}{\mathbb{Z}_2}$

$$\boxed{ds^2 = f(r)\, dr^2 + a(r)\, \sigma_1^{\,2} + b(r)\, \sigma_2^{\,2} + c(r)\, \sigma_3^{\,2}}$$

f, a, b, c

$$f = -\frac{b(r)}{r}$$

$$\begin{cases} a^2 = r^2\left(1 - \frac{2}{r}\right) - 8 \\ b = \cdots \\ c = \cdots \end{cases}$$

elliptic integrals.

- 3 monopoles in $SU(2)$.

Electric magnetic duality

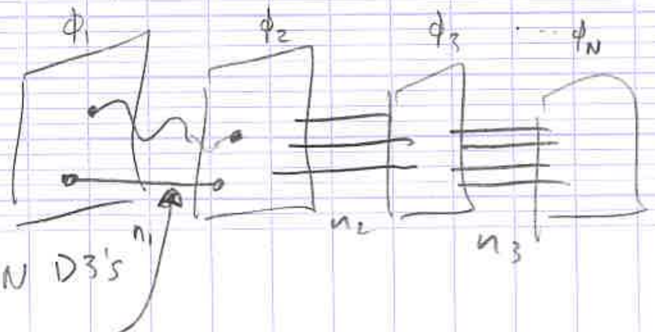
D-branes and monopoles:

(Nahm)

Nahm's equations:

$\mathcal{N} = 4$ SYM $\quad U(N)$

$\langle\phi\rangle = \mathrm{diag}(\phi_1, \dots, \phi_N]$



D1-strings. ≡ monopoles.

$\hat{g} = \sum' n_a \vec{\alpha}_a$

What is worldvolume dynamics on D1 strings.

Hmmm.... $\prod_{a=1}^{N-1} U(n_a)$ with each living on some

interval $\phi_a \leq s \leq \phi_{a+1}$

(SUSY w/ impurities)

(Sethi/Kapustin
D. Freedman.),

$$T_2 \left(\mathcal{D}_s X^i - \frac{i}{2} \epsilon^{ijk} [X^j, X^k] + \text{impurities} \right)^2$$

$i = 1, 2, 3$

tangent to D1

6/220/07

Intersecting D-branes & Particle Physics

w/Blumenhagen hep-th/0502005

"Modern" perspective of string phenomenology

from D-branes:

1) Non abelian gauge symmetries

$SU(3)_c \times SU(2)_L \times U(1)_Y$

2) Chiral matter

$Q_L = (3, 2, 1/6)$ $\quad Q = T_{3L} + Y$

families (in 3 copies)

$l_L = (1, 2, -1/2)$

$u_R = (3, 1, -1/3)$

$d_R = (3, 1, -1/3)$

$l_R = (1, 1, -1)$

$\nu_R = (1, 1, 0)$

$H_{u,d} = (1, 2, \pm 1/2)$ SSB of EW

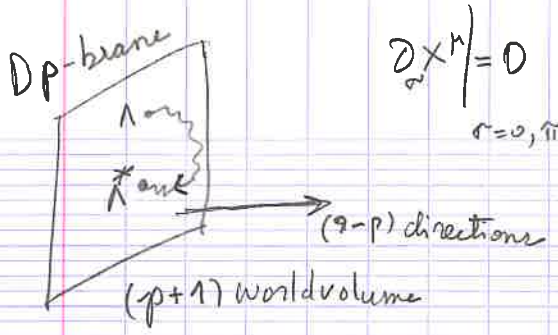
3) 3 families

Geometric origin of 1) - 3)

Couplings ~~a~~

$\frac{1}{g^2_{YM}} Tr(F^2)$ gauge etc.

$h_u Q_L u_R H_u$ Yukawa etc.



$\partial_\sigma X^\mu \Big|_{\sigma=0,\pi} = 0$ Neumann b.c $\mu = 0, \dots, p$

$\partial_\tau X^\mu \Big| = 0$ $\mu = p+1, \dots, 9$ Dirichlet

$$\boxed{\xi_\mu \partial_z X^\mu e^{ik_\mu X^\mu}}$$ spin 1 field $U(1)$

polarization

dim 1

Λ, Λ^* "annihilate"

- $k_\mu^2 = 0$ massless
- spin 1

Adding more // ~~gra~~ branes : N coincident

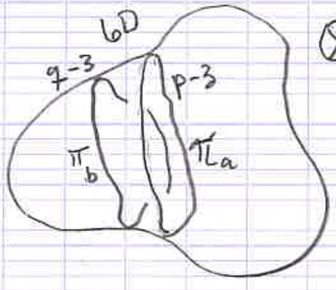
$i, j = 1, \dots, N$ Λ_i, Λ_i^*

vertex operator has same form

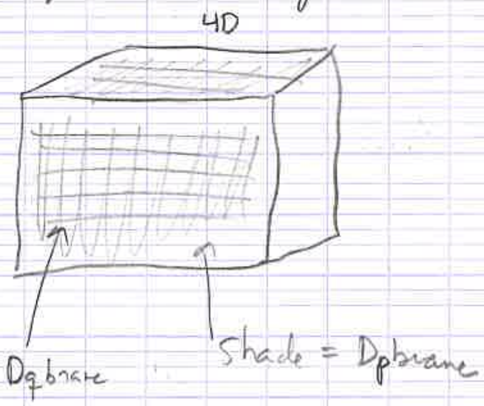
$$\xi_\mu \partial_z X^\mu e^{ik_\mu X^\mu} \underbrace{\Lambda_i \otimes \Lambda_j^*}_{\text{adjoint of } U(N).}$$

<u>Compactification and getting chirality from D-brane picture:</u>

$\mu = 0, \dots, 9$ $10 \xrightarrow[\text{compactify 6}]{} \boxed{4}$



$\otimes$



Calabi-Yau
special curvature or Holonomy.

mostly deal with Toroidal Orbifolds

Rich Structure!

- singularities
- intersecting branes
- Magnetized branes

Focus → D6-brane

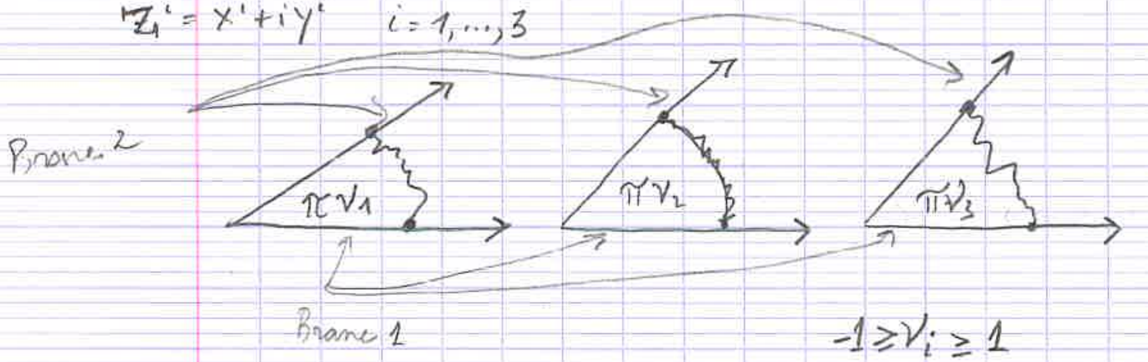
⇒ $p-3 = 3$ cycles are wrapped.

intersect at finite # of points

What happens to quantized strings here?

locally 3 complex coords. locally flat

$Z_1^i = x^i + iy^i \qquad i = 1, \dots, 3$



$-1 \geq \nu_i \geq 1$

B.C.'s

$$\partial_\sigma X^i\Big|_{\sigma=0} = \partial_\tau Y^i = 0 \qquad \text{Brane 1}$$

$$\partial_\sigma\left(\cos(\pi\nu_i)X^i + \sin(\pi\nu_i)Y^i\right)\Big|_{\sigma=\pi} = 0 \qquad \text{Brane 2}$$

$$\partial_\tau\left(-\sin(\pi\nu_i)X^i + \cos(\pi\nu_i)Y^i\right)\Big|_{\sigma=\pi} = 0$$

Still free string theory, but not integer moded:

Mode Expansion:

$$z = e^{(i\sigma + \tau_E)} \qquad \sigma = 0 \qquad \partial_{(z \mp \bar{z})} \left(Z^i \pm \bar{Z}^i \right) = 0$$

$$\sigma = \pi \qquad e^{-i\nu\pi} \partial_{(z \mp z)} Z^i + e^{+i\nu\pi} \partial_{(z \mp \bar{z})} \bar{Z}^i = 0$$

$\nearrow$

B.C.'s in terms of z's.

$$Z^i = \sum_{n \in \mathbb{Z}}{}' \frac{\alpha_{n-\nu^i}}{n-\nu^i} z^{-(n-\nu_i)} + \frac{\tilde{\alpha}_{n+\nu^i}}{n+\nu^i} \bar{z}^{-(n+\nu_i)}$$

$$\bar{Z}^i = \sum_{n \in \mathbb{Z}}{}' \frac{\bar{\alpha}_{n-\nu_i}}{n-\nu_i} \bar{z}^{-(n-\nu_i)} + \frac{\bar{\tilde{\alpha}}_{n+\nu_i}}{n+\nu_i} z^{-(n+\nu_i)}$$

$$\alpha_{n-\nu_i} = \bar{\alpha}_{n-\nu_i} \qquad \tilde{\alpha}_{n+\nu_i} = \bar{\tilde{\alpha}}_{n+\nu_i}$$

Doubling trick:

$$\mathrm{Im}\, z > 0 \qquad Z^i = \sum_{n \in \mathbb{Z}}{}' \frac{\alpha_{n-\nu_i}}{n-\nu_i} z^{-(n-\nu_i)}$$

$$\bar{Z}^i = \sum_{n \in \mathbb{Z}}{}' \frac{\tilde{\alpha}_{n+\nu_i}}{n+\nu_i} z^{-(n+\nu_i)}$$

$$\mathrm{Im}\, z < 0 \qquad Z^i = \sum \frac{\tilde{\alpha}_{n+\nu_i}}{n+\nu_i} z^{-(n+\nu_i)}$$

$$\bar{Z}^i = \sum{}' \frac{\alpha_{n-\nu_i}}{n-\nu_i} z^{-(n-\nu_i)}$$

strings are stuck at intersections, twisted sector.

→ twisted vacuum.

$|\sigma^i\rangle$

$\uparrow$ bosonic twist field

scaling dimension – $\boxed{\tfrac{1}{2}\nu_i(1-\nu_i) = h_i}$

Go through same thing with fermions:

$$\Psi^i_{(z)} = \sum_{\substack{r = \mathbb{Z}\ (R) \\ r = \mathbb{Z} + 1/2\ (NS)}}' \psi^i_{r-\nu_i}\, z^{-(r-\nu_i+\frac{1}{2})}$$

$$\Psi^i(\bar{z}) = \sum_{\substack{r\in\mathbb{Z}(R) \\ r\in\mathbb{Z}+\frac{1}{2}\ (NS)}}' \tilde{\psi}^i_{r+\nu_i}\, \bar{z}^{-(r+\nu_i+\frac{1}{2})}$$

again <u>Doubling</u> <u>Trick</u>

$\mathrm{Im}\, z > 0$ → same as above

$\mathrm{Im}\, z < 0$

Bosonization. Usually $e^{i\frac{1}{2}H^i}$

Now,

$$\left.\begin{array}{ll} e^{i(\nu_i - 1/2)H^i} & \text{– R-sector} \\ e^{i(\nu_i - 1)H^i} & \text{– NS-sector} \end{array}\right\}$$

~~Spi~~ spin fields

<u>Let's write physical states:</u>

<u>Focus on spacetime fermions</u>

<u>R-sector</u> → spacetime fermion

$$\underset{\uparrow}{e^{-\phi/2}}\; e^{i(\pm h_1\cdot\frac{1}{2} \pm h_2\cdot\frac{1}{2})} \prod_{i=1}^{3} e^{i(\nu_i-\frac{1}{2})H_i}\, \rho^i\, e^{ik_\mu X^\mu}\, \Lambda_a \tilde{\Lambda}_b^{*}$$

superconformal ghosts

"1/2 picture"

What is conformal dimension?

$$\left.\begin{aligned} e^{a\phi} &\rightarrow h_a = \frac{-a(a+2)}{2} \\ e^{b_i H_i} &\rightarrow \frac{b_i^2}{2} \end{aligned}\right\} \text{conformal dimension.} \qquad (\alpha' = 2)$$

$$\frac{3}{8} + \frac{1}{8} + \frac{1}{8} + \sum_{i=1}^{3} \left[\frac{(\nu_i - \frac{1}{2})^2}{2} + \frac{1}{2}\nu_i(1-\nu_i) \right]$$

$$= \frac{1}{8}(3 + 1 + 1 + 3) = 1. \quad \checkmark$$

$\Rightarrow$ Massless physical states, no ν_i dependence
Chan paton factors $\rightarrow$ $U(N_a) \times U(N_b)$

NS-sector $\rightarrow$ spacetime bosons

(-1 picture)

$$e^{-\phi} \prod_{i=1}^{3} e^{i(\nu_i - 1)H^i} \sim^{i} e^{ik_\mu X^\mu}$$

$$\frac{1}{2} + \sum_i \left(\frac{(\nu_i - 1)^2}{2} + \frac{1}{2}\nu_i(1-\nu_i) \right) - \frac{1}{2}\sum' \nu_i = 2 - \frac{1}{2}\sum \nu_i = h$$

if $h < 0$ -> tachyon
$h = 0$ -> massless
$h > 0$ -> massive

$\boxed{\sum \nu_i = 1 \Rightarrow \text{massless}}$ $\Rightarrow$

$\sum \nu_i > 1 \Rightarrow$ tachyon

$\sum \nu_i < 1 \Rightarrow$ massive

6/22/05

Hewitt II

Parton Model :

(Feynmann & Bjorken)

quarks & gluons → partons

1) all partons interact independently

2) fractional ‸ momenta carried by parton
longitudinal

$$x_i P_{\text{proton}} = \underset{\substack{\uparrow \\ \text{parton}}}{P_i} \qquad x_i \in [0,1]$$

$$\sum_i P_i = P_{\text{proton}}$$

3) Hard scattering occurs at the parton level
calculated perturbatively in QCD, EWK

4) Nonperturbative junk → $\boxed{\text{parton distribution}}$ function
= probability of extracting parton w/x_i parametrize
all nonperturbative QCD

determined by global fit to all data: CTEQ
MRST

these might change @ LHC energies.

shapes:

$$f_{\text{VALENCE QUARKS}} \sim (1-x)^3 \; ; \; f_{sea} \sim \frac{1}{x}(1-x)^8$$

$$f_{\text{gluons}} \sim \frac{1}{x}(1-x)^5$$

$f(x, Q^2)$ ← energy transfer in a reaction.

5) total cross section

$$\sigma(AB \to x) = \sum_{\text{partons}}' dx_a dx_b \, f_a^{(A)}(x_a, Q^2)\, f_b^{(B)}(x_b, Q^2)\, \hat{\sigma}(ab \to x)$$

↑ observed

CTEQ
MRST

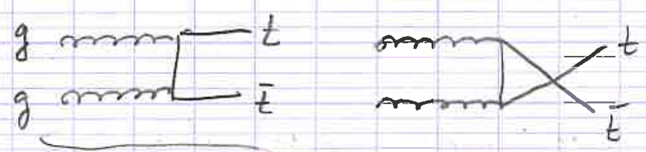
↑ calculated

hat: $\hat{\sigma}$ denotes parton frame.

$\hat{s} = x_a x_b s \equiv \tau s$ (CM energy in parton frame)

Ex: Top pair production

top quark → gluons



gluon fusion

$q\bar{q}$ annihilation

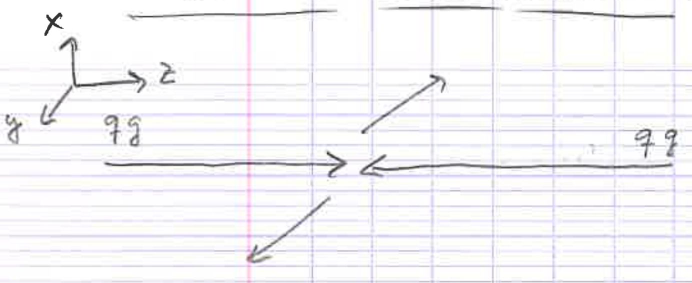
↑ important @ LHC

$$\sigma(p\overset{(-)}{p} \to t\bar{t}) = \int dx_a dx_b \left\{ f_g^{p}(x_a, Q^2)\, f_g^{\overset{(-)}{p}}(x_b Q^2)\, \hat{\sigma}(gg \to t\bar{t}) \right.$$

$$+ \left[f_g^{p}(x_a, Q^2)\, f_{\bar{q}}^{\overset{(-)}{p}}(x_b, Q^2) + f_{\bar{q}}^{p}(x_a, Q^2)\, f^{\overset{(-)}{p}}(x_b, Q^2) \right]$$

$$\times \hat{\sigma}(q\bar{q} \to t\bar{t})$$

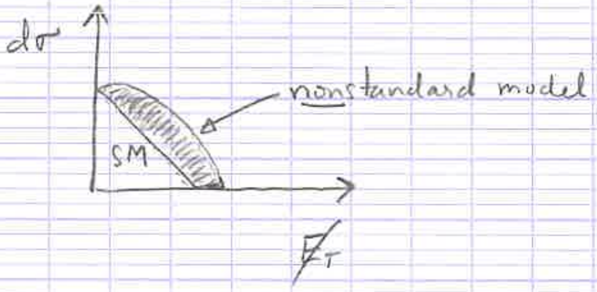
Hadron Collider variables:



NO longitudinal information.

1) $p_T = \sqrt{p_x^2 + p_y^2}$; $p_T^* = p_T$ (

$\not{E}_T$ = missing tranverse energy



2) Rapidity (related to scattering angle

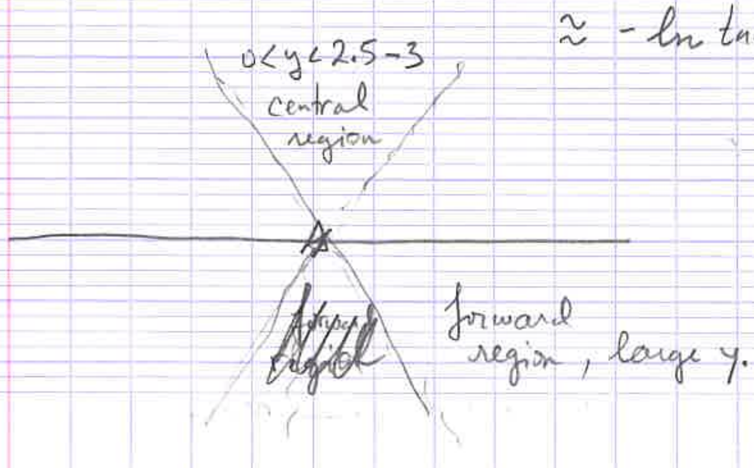
$$y = \frac{1}{2} \ln \left[\frac{E^{cm} + p_z^{cm}}{E^{cm} - p_z^{cm}} \right]$$

$$= \frac{1}{2} \ln \left[\frac{1+\beta}{1-\beta} \right] = \frac{1}{2} \ln \left[\frac{x^a}{x^b} \right]$$

y is boost invariant $y \in \mathbb{R}$

~~F~~ For $p \gg m$ $y = \frac{1}{2} \ln \left[\frac{\cos^2(\theta/2) + m^2/4p}{\sin^2(\theta/2) + m^2/4p} \right.$

$\simeq -\ln \tan \theta/2 = \eta$ = pseudo-rapidity



Ex: Standard model Higgs production

SM: SSB, 1 Higgs doublet, neutral scalar w/ $J^{\pi} = 0^{+}$

$$\downarrow$$

$$\phi$$

$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$ SSB: shift $\chi = \phi - v$

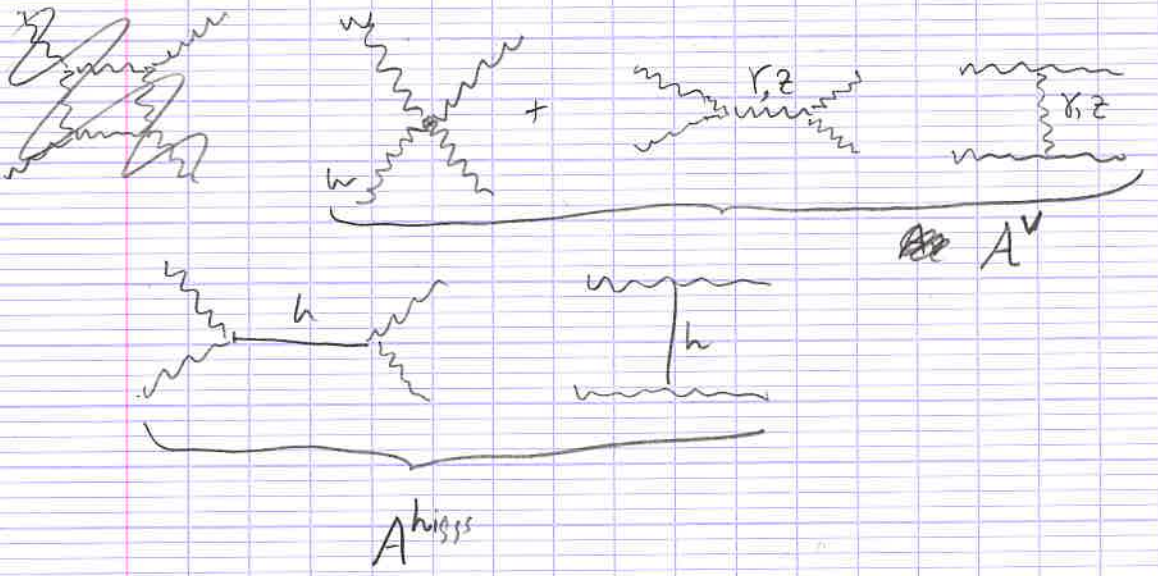
$$\begin{cases} V(\chi) = \frac{1}{2} m_{\chi}^2 \chi^2 + \lambda v \chi^3 + \frac{1}{4}\lambda \chi^4 \\ m_{\chi} = \sqrt{2}\lambda v \sim 348\, \lambda^{1/2}\ \text{GeV} \end{cases}$$

Must be tested @ LHC.

HUGE TEST

Unitarity in gauge boson scattering: Lee, Quigg, Thacker '77

WW → WW ($2 \rightarrow 2$ scattering) @ high energies



Look at $s \gg m_W^2$

growing with energy.

$\mathcal{A}^V \sim -\frac{s-t}{M_W^2} + \frac{2t}{s}\left(\frac{m_Z^2}{m_W^2} - 4\right)$ + much more

$\mathcal{A}^{Higgs} \sim \frac{t+s}{M_H^2}$ + other stuff

$s < m_h^2 \Rightarrow A^V$ dominates, violate perturbative unitarity (optical theorem) at $\sqrt{s} = 1.7$ TeV

$$s > m_h^2 \quad A^{TOT} \sim \frac{m_Z^2}{m_W^2}\left(1+\frac{s}{t}+\frac{t}{s}\right) + \frac{m_h^2}{m_W^2} - i m_h \Gamma_h$$

If no higgs ⇒ blow up scattering in WW → WW (NO LOSE THM)

$$m_H < \left(\frac{8\pi\sqrt{2}}{3G_F}\right)^{1/2} \sim 800 - 1000 \text{ GeV}$$

Higgs production + Decay is governed by its couplings

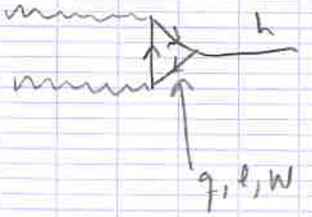
1) $f\bar{f}h$ $\quad \mathcal{L} \sim \underbrace{-(\sqrt{2}G_F)^{1/2}\, m_f}_{\text{small} = \frac{m_f}{246}} f\bar{f}h$

small except for tops.

2) VVh $\quad \mathcal{L} \sim (\sqrt{2}G_F)^{1/2}\left(2m_W^2 h W_\mu^+ W^{-\mu} + m_Z^2 h Z_\mu Z^\mu\right)$

(much larger than 1))

3) $\gamma\gamma h$ couples through loops



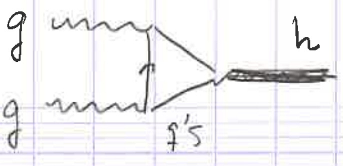
new stuff

$$\mathcal{L} \sim -\frac{\alpha}{2\pi}(\sqrt{2}G_F)^{1/2}\, \underset{\uparrow}{I}\, F_{\mu\nu}F^{\mu\nu} h$$

loop integral

in SM → tops & W's dominate but with opposite sign.

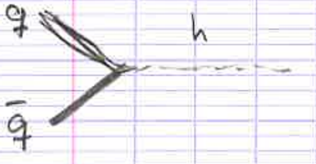
4) ggh



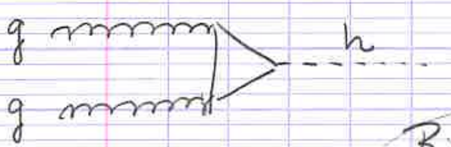
(top dominates, but tops & stops mostly cancel off)

$$\mathcal{L}_{eff} \sim \frac{\alpha_s(m_h^2)}{12\pi}(\sqrt{2}G_F)^{1/2}\, I\, G^a_{\mu\nu}G'^{\mu\nu}_a\, h$$

Production mechanisms @ LHC



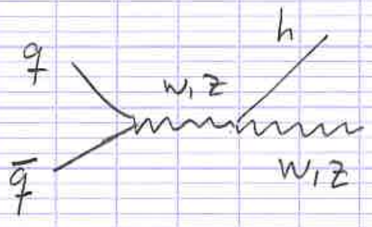
X too small.



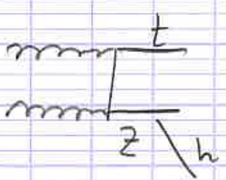
(gluon fusion production mechanism)

Biggest dσ X

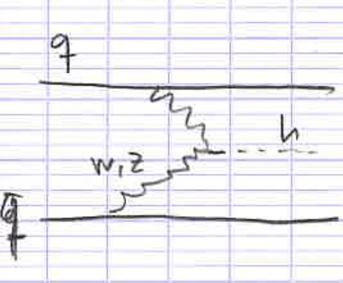
others



Wh, Zh



$t\bar{t}h$



(Vector boson fusion)

see 2 forward jets.
↕
"tag"

Hewitt III

$gg \to h \to \gamma\gamma$ · ATLAS Techical Design Report

Higgs mass resolution ~ 1%

$\sigma \sim (pp \to h \to \gamma\gamma) \sim 4pb$

Exp't: (Cuts that isolate signal.)

γ candidates – blobs in calorimeter $p_T^1 > p_T^2$

$p_T^1 = 40 GeV$ $p_T^2 > 25 GeV$ } ← art. designed to isolate signal.

$|\eta_{1,2}| < 2.5$ pseudo rapidity. ⇒ (central region).

$$\frac{p_T^1}{(p_T^1 + p_T^2)} < 0.7 \quad \Big\} \text{ cuts background}$$

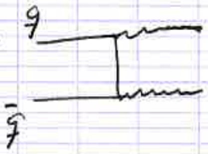
Acceptance: How much is left after cuts. ~ 40%

Efficiency: Identification & Deconstruction ~ 80%

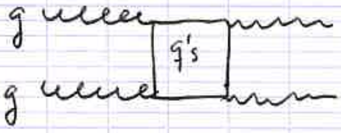
Backrounds:

Irreducible (production of genuine photon pairs)

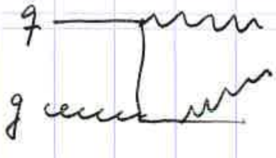
1. Born process $q\bar{q} \to \gamma\gamma$



2. Box $gg \to \gamma\gamma$



3. Brem. $qg \to q\gamma \to q\gamma\gamma$



$\sigma_{\text{background}} \sim 1\,\text{pb}/\text{GeV}$ $m_{\gamma\gamma}$

reduced 50% by isolation cuts

<u>Reducible</u>: (mis identified photons)

1) jet-jet $\sigma_{jj} \sim 2\times10^{6}\,\sigma_{\gamma\gamma}$ (so 1 misidentification in a million is a problem).

2) γ-jet $\sigma_{\gamma j} \sim 8\times10^{2}\,\sigma_{\gamma\gamma}$

3) $Z \to ee$

Rejection factors of order $\underset{1)}{10^{7}}$, $\underset{2)}{10^{3}}$

$$\left.\begin{array}{l}\cdot\ \text{Requirements on leakage:}\\ \cdot\ \text{calorimeter isolation}\\ \cdot\ \text{rms width of shower in em calorimeter}\end{array}\right\} \leftarrow \begin{array}{l}\text{Rejection factors.}\\ \text{(understand detector)}\end{array}$$

~~ar~~ Reducible background is down to size of irreducible background.

$m_h =$	120	130	150
$\sigma \times BR$ (fb)	51	45	24
Acceptance × eff	29%	30%	33%
signal events in mass bin	1040	950	560
bkrnd. in mass bin $\gamma\gamma$-	26,300	22,600	15,300
jet - jet	1200	1200	900
γ - jet	3200	2700	1800
	5.9	4.8	4.2

(≥ 5 is a discovery).

Statistical significance : $\text{Signal}/\sqrt{\text{background}}$

Theory Status : $gg \to h$

full NLO (next to leading order) complete.
used to do $m_t \to \infty$. Now we ~~ac~~ use actual value of m_t. + agrees with $m_t \to \infty$.

$$+ \underset{\underset{\log.}{\uparrow}}{\text{NNLL}}$$

NNLO — use $m_t \to \infty$.

$\Rightarrow$ 10% theory error in σ. (would be good to improve to 5%).

for $m_h > 150$ $gg \to h \to ZZ^* \to \underline{\underline{4\ell\text{'s}}}$ "gold plate mode"

Higgs Parameters~~s~~ determination.

1. Mass: accurately determined from ZZ channel. (LHC).

$$\Rightarrow \frac{\Delta m_h}{m_h} \lesssim 10^{-3}$$

2. Total width: from width of peak in ZZ, large errors
5% ~ 100%

3) Couplings: (really important for what type of Higgs). (LHC
doesn't do so well.)

in model independent way, we can get ratios
of couplings to 10% ~ 20% level

model dependent: $\boxed{\text{fix } \Gamma_b / \Gamma_T}$ to SM value
$\Gamma_{W,Z,\tau}$

$$\uparrow$$

holds for SUSY Higgs
& radion – Higgs mixing

$$\sigma(h)\,BR(h\to xx) = \frac{\boxed{\sigma(h)^{SM}}}{\Gamma^{SM}_{initial}} \cdot \frac{\Gamma_{initial}\,\Gamma_x}{\Gamma}$$

uncertainty in pdf causes uncertainty in this.

observed channels:

$gg \to h \to \gamma\gamma, ZZ, WW$

$WH \to WWW \to W\gamma\gamma$

$Zh \to Z\gamma\gamma$

$\boxed{t\bar{t}h \to h \to WW, \gamma\gamma, b\bar{b}}$ ← difficult to pull out.

$qqh \to qqWW, ZZ, \gamma\gamma, ZZ$

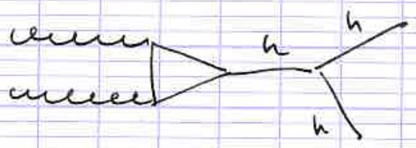
take ratios:

$$\frac{\sigma(gg\to h)\,BR(h\to\gamma\gamma)}{\sigma(gg\to h)\,BR(h\to ZZ^*)} = \frac{\Gamma_\gamma}{\Gamma_Z}$$

4) Higgs self-coupling

$$V(\chi) = \frac{1}{2}m_\chi^2\chi^2 + \lambda v\chi^3 + \underbrace{\frac{1}{4}\lambda\chi^4}$$ in Standard Model.

need upgrade of LHC to measure this.



$b\bar{b}\gamma\gamma$ for $m_h < 140\,GeV$

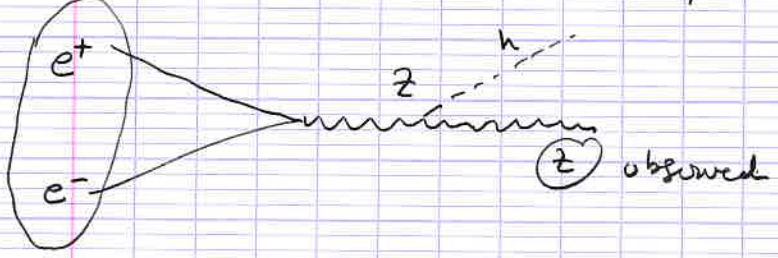
$W^{(*)}WW^{(*)}W$ for $m_h > 140\,GeV$.

w $600\,fb^{-1}$
$-1.1 < \Delta\lambda < 1.6$
$\Delta\lambda = \frac{\lambda}{\lambda_{SM}} - 1$

can show $\lambda \neq 0$ at 95% CL for $150 \le m_h \le 200$

LHC upgrade $3\,ab^{-1}$ λ determined to 20%-30%

Compare with Linear collider: Higgs production at linear collider



Known

$$m_{recoil} = \sqrt{s - 2\sqrt{s}\,E_{f\bar{f}} + m^2_{f\bar{f}}}$$

isolates the Higgs state no matter how it decays.

This is it!

Also, we get an accurate and direct determination of Higgs properties in a complete, model independent fashion.

Level of accuracy ~ few %

6/23/05 Cvetic

I. Non-abelian gauge symmetry – N coincident D-branes

II. $\sum_{i=1}^{3} \pi \nu_i = 2\pi \Rightarrow m^2 = 0$ Chirality $U(N_a) \times U(N_b)$
$(\Box_a , \bar{\Box}_b)$

III. Family Replication – compact space has pot'l to produce finite # of intersection points.

$[\Pi_a] \circ [\Pi_b]$ – topological number

Engineering standard model:

$N_a = 3$; Π_a $U(3)_c \times U(2)_L \times \boxed{U(1)_Y}$ (∃ anomaly issue ... c.f. Uranga)

$N_b = 2$; Π_b $(\underline{3} , \underline{2}) \sim Q_L$

$[\Pi_a] \circ [\Pi_b] = 3 \Rightarrow$ 3 families of left handed quarks.

Pati-Salam symmetry

$U(4)_{PS} \times U(2)_L \times U(2)_R$

	$N_a = 4$	$N_b = 2$	$N_c = 2$
$\begin{pmatrix} Q_L \\ L_L \end{pmatrix}$	$\underline{4}$	$\bar{\underline{2}}$	$\underline{1}$
$\begin{pmatrix} Q_R \\ L_R \end{pmatrix}$	$\bar{\underline{4}}$	$\underline{1}$	$\underline{2}$
(H_{uR})	$\underline{1}$	$\underline{2}$	$\bar{\underline{2}}$

$[\Pi_a] \cdot [\Pi_b] = 3$

$[\Pi_a] \circ [\Pi_c] = -3$

$[\Pi_b] \cdot [\Pi_c] =$ # of Higgs.

seems to be large in some models. ~ 12

1. Global Consistency conditions

   · Total D6-charge in internal space = 0

2. Supersymmetry condition

Plus Examples of standard models and their features (Toroidal Orbifolds)

Also, Adding flux.

Slides —

Supersymmetry conditions

3 cycles - Special Lagrangian manifolds

$(1,1)$ J $\nabla J = 0$ Kahler form

$(3,0)$ Ω $\nabla \Omega = 0$ Kahler pot'l

Lagrangian $\Longleftrightarrow$ $J\Big|_{\Pi^a} = 0$

Special $\Longleftrightarrow$ $\mathrm{Im}\,\Omega\Big|_{\Pi_a} = 0$

$$V = \int \mathrm{Re}\,\Omega\Big|_{\Pi_a}$$

$$J = i\sum_{i=1}^{3} dz^i \wedge d\bar{z}^i \qquad \Omega = dz^1\wedge dz^2 \wedge dz^3$$

$$= -2\sum dx^i\wedge dy^i$$

$dy^i = dx^i \tan(\pi\nu_i) \rightarrow J\Big|_{\Pi} = 0$ obviously

$$\mathrm{Im}\,\Omega = -i\left(dy^1\wedge dy^2\wedge dy^3 - dy^1\wedge dx^2\wedge dx^3 + \text{cyclic}\right)$$

$$= \left(\sim\sim\sim\sim\right) dx^1\wedge dx^2 \wedge dx^3$$

$$\downarrow$$

$$\left\{\prod_1^3 \tan(\pi\nu_i) - \sum_{i=1}^{3}\tan(\pi\nu_i)\right\} = 0$$

Equivalent $\sin\left(\sum_1^3 \pi \nu_i\right) = 0 \Rightarrow \boxed{\sum_{i=1}^{3} \pi \nu_i = 0 \bmod \cancel{2}\pi.}$

BUT... with the condition of positive volume :

$$\cos\left(\sum_{i=1}^{3} \pi \nu_i\right) \geqslant 0 \Rightarrow \boxed{\sum_i \pi \nu_i = 0 \bmod 2\pi}$$

Slides again —

Tong 3: Vortices

- Increase $SU(N) \to U(N)$
- Add Matter in the fundamental representation

Pick N_f scalar fields q_i $i=1,\dots,N_f$

$$S = \int d^4x \; \mathrm{Tr}\left[\frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2e^2} D_\mu \phi\right] + \sum_{i=1}^{N_f} |\mathcal{D} q_i|^2$$

$$- \sum_{i=1}^{N_f} q_i^\dagger \phi^2 q_i - \frac{e^2}{2} \mathrm{Tr}\left[\sum_{i=1}^{N_f} q_i q_i^\dagger - v^2 \mathbb{1}_N\right]$$

FI term.

tensor product, rank N_f matrix

rank N

$\to v^2 \int \mathrm{Tr}(V)$

For the first part of lecture $N_f = N$

Unique vacuum state: $\phi = 0$ $q_i^a = \delta_i^a v$ $a=1,\dots,N$ color

$i=1,\dots,N$ flavour

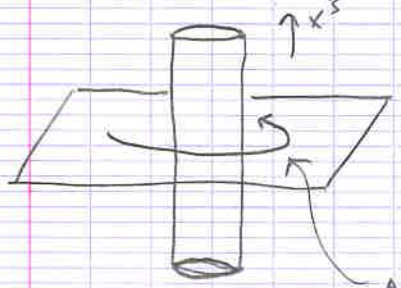
This vacuum has a mass gap

$$m_\gamma^2 = m_\phi^2 = m_q^2 = e^2 v^2$$

Symmetries:

$$U(N)_{gauge} \times SU(N)_{flavour} \longrightarrow SU(N)_{diag}$$

with $q \to u q V^\dagger$ "color flavour locking"



winding classified by

$$\pi_1\left(U(N) \times SU(N) / SU(N)_{diag} \right) \cong \mathbb{Z}$$

phase of q winds at ∞

Why does winding $q \Rightarrow$ magnetic field B?

$$\mathcal{D}q \to 1/r^2 \quad \text{as } r \to \infty$$

but $\partial_\theta q \sim 1/r \Rightarrow A_\theta \sim \frac{1}{r} \Rightarrow B_3 \neq 0$

need $\mathcal{D}_\theta q = \partial_\theta q - iA_\theta q \to 0 \Rightarrow A_\theta \to i\partial_\theta q \cdot q^{-1}$ as $r \to \infty$

The winding number $k \in \mathbb{Z}$

$$2\pi k = \mathrm{Tr} \oint_{S^1} i\partial_\theta q q^{-1} = \mathrm{Tr} \oint_{S^1} A_\theta = \mathrm{Tr} \int dx^1 dx^2 B_3$$

<u>Vortex Equations</u> set $\left\{ \begin{matrix} \partial_0 = \partial_3 = 0 \\ A_0 = A_3 = 0 \end{matrix} \quad \phi = 0 \right\}$

Tension of Vortex $T_{\text{Vortex}} = \int dx^1 dx^2 \,\mathrm{Tr}\left[\frac{1}{2e^2} B_3^2 + \frac{e^2}{2} \left(\sum_i q_i q_i^\dagger - v^2 \right)^2 \right.$

$$+ \left| \mathcal{D}_r q_i \right|^2$$

$$= \int dx^1 dx^2 \frac{1}{2e^2} \mathrm{T}_2 \left[B_3 - e^2 \left(\sum_i q_i q_i - v^2 \right) \right]^2 + \left| \mathcal{D}_1 q_i + i \mathcal{D}_2 q_i \right|^2$$

~~$\mathrm{Tr}\,B_3$~~ $- v^2 \int d^2x \,\mathrm{T}_2 B_3$

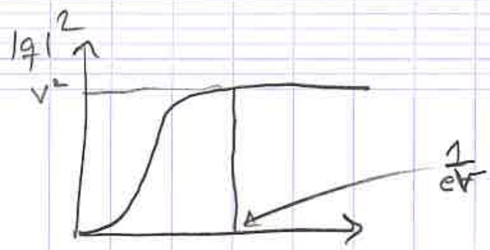
$[\mathcal{D}_1, \mathcal{D}_2] = -iF_{12}$
$= -iB_3$

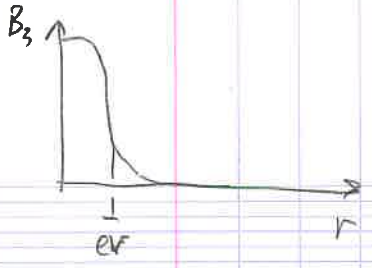
$$\geq -v^2 \int d^2x \,\mathrm{Tr} B_3 = 2\pi v^2 |k|$$

Vortex equations:

$$\boxed{\begin{matrix} B_3 = e^2 \left(\sum_i q_i q_i - v^2 \right) \\ \mathcal{D}_{\bar{z}} q_1 = 0 \end{matrix} \quad z = x^1 + ix^2}$$

- No closed sol^n is known





U(N) vortex solution:

U(1) vortex soln U(1) vortex soln.

$$A_\mu = \begin{pmatrix} A_\mu^{*} & & \\ & 0 & \\ & & \ddots & \\ & & & 0 \end{pmatrix} \qquad q = \underbrace{\begin{pmatrix} q^{*} & & & & \\ & v & & & \\ & & v & & \\ & & & \ddots & \\ & & & & v \end{pmatrix}}_{\text{flavour}} \Bigg\} \text{color}$$

$\Rightarrow$ oriented zero modes $SU(N)_{\text{diag}} \big/ S[U(N-1) \times U(1)] \cong \mathbb{C}P^{N-1}$

The Vortex Moduli Space:

Define $V_{k,N}$ $\quad \boxed{\dim V_{k,N} = 2Nk}$ k parallel vortex strings

\# vortices U(N) gauge group. Index thm

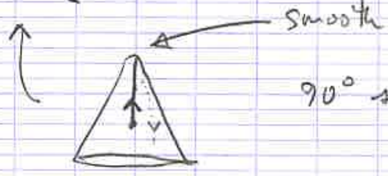
Natural metric:

- Kahler
- smooth (no singularities)
- $SU(N) \times U(1)$ isometry.
- Unknown for $k \geq 2$.

Examples:

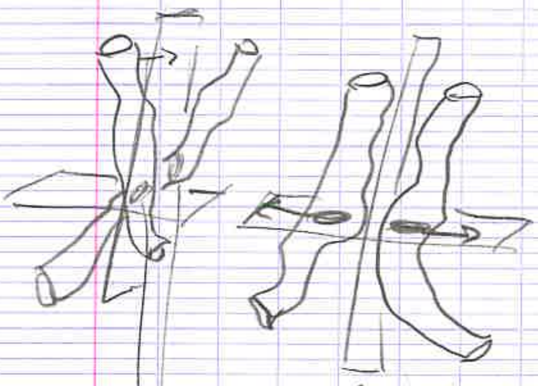
- $V_{1,N} \cong \mathbb{C} \times \mathbb{CP}^{N-1}$ Kahler class $\boxed{r = \frac{2\pi}{e^2}}$

- $V_{2,1} \cong \mathbb{C} \times \mathbb{C}/\mathbb{Z}_2$



90° scattering.

$\boxed{\text{90° scattering for monopoles too}}$

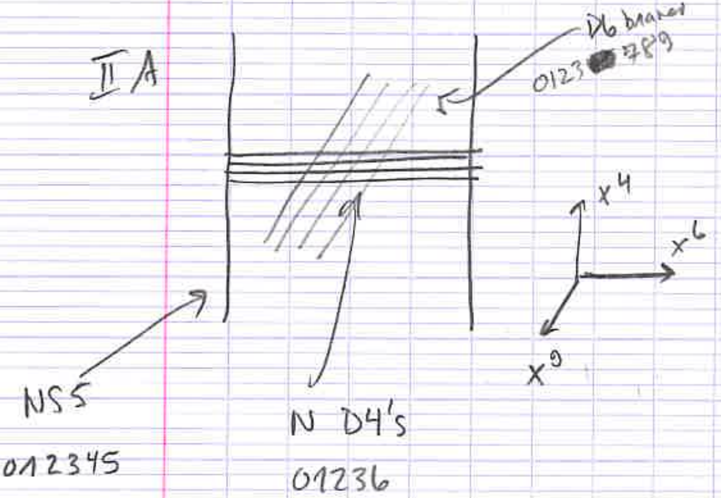


← cosmic string <u>always</u> interact this way. (F-string interact w/probability gs)

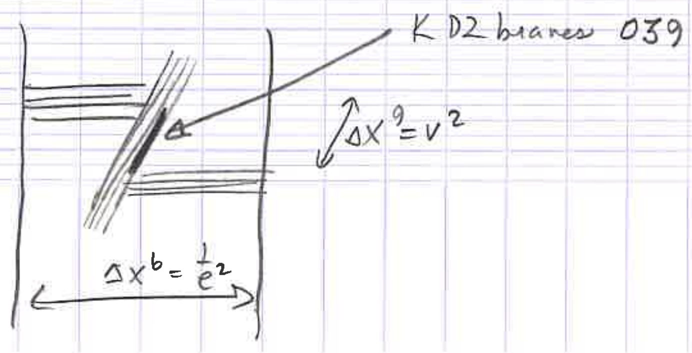
D-branes and Vortices:

d = 3+1 U(N) SYM with N = 2 + N flavours.

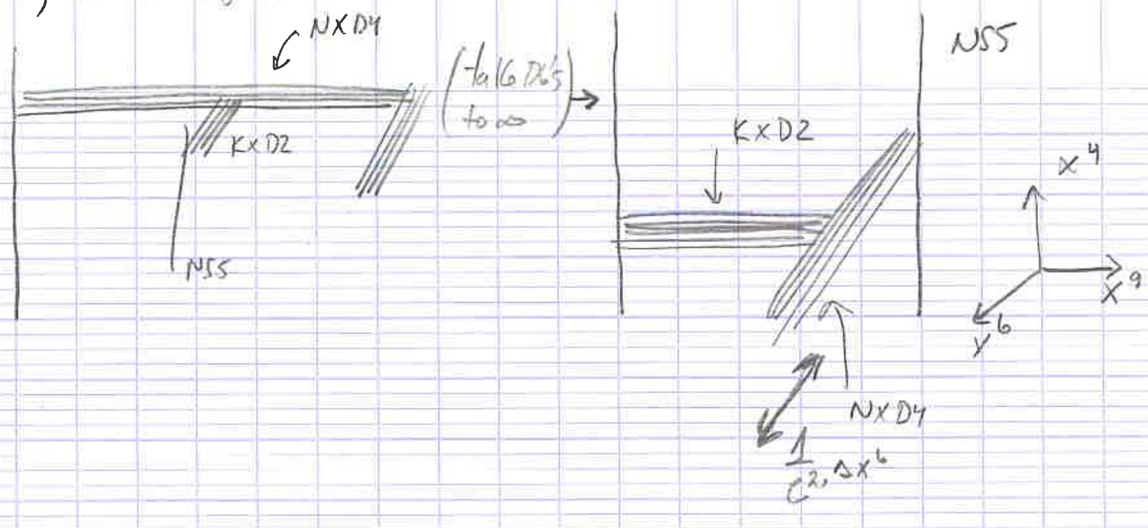
IIA



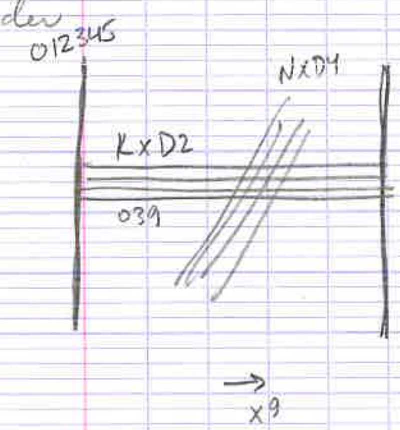
Pull right brane out of board.



Move D6's in x^6, to the right.



Consider



$d = 1+1$

$U(K)$ gauge theory

$\sigma = x^4 + i x^5$

$Z = x^1 + i x^2$

$+ N$ hypermultiplets $\psi_i, \tilde{\psi}_i$

with couplings $\psi_i^{\dagger} \sigma^2 \psi + \tilde{\psi} \sigma^2 \tilde{\psi}^{\dagger}$

The theory on the vortex string is:

$d = 1+1$, $\mathcal{N} = (2,2)$ SUSY ; $U(k)$ gauge thy. + adjoint chiral Z + N fundamental chirals ψ_i

+ FI parameter $r = \frac{2\pi}{e^2}$

$$V = \mathrm{Tr} |[\sigma, Z]|^2 + \sum_{i=1}^{N} \psi_i^{\dagger} \sigma^2 \psi_i + \frac{g^2}{2} \mathrm{Tr} \left(\sum_{i=1}^{N} \psi_i \psi_i^{\dagger} + [Z, Z^{\dagger}] - r \right)^2$$

$g^2 \to \infty$

$$\mathcal{M}_{Higgs} \cong \{V = 0\} / U(k) \overset{!}{\cong} \mathcal{V}_{k,N}$$

Examples: $V_{1,N}$:

z decouples

$$\{\sum |\phi_i|^2 = r\} \Big/ U(1) \cong \mathbb{CP}^{N-1}$$

Metric on $V_{K,N}$?
Sol'ns to vortex eqns
v.s. ~~vortex~~ soln to
$V=0$ eqns.

Vortex theory $U(k)$ + adjoint chiral + N fundamental + N fundamental chirals + FI $= r$ } $V_{K,N}$

Instanton Thy $U(K)$ + adjoint hyper + N fundamental hypers } $\mathcal{L}_{K,N}$

$V_{K,N} \cong \mathcal{L}_{K,N}\Big|_{\hbar=0}$ ϵ — rotation of instantons in x^3-x^4 plane.

Why?

Look at $k=1$ vortex

$U(1)$ + N charged chirals w/ $\left(\sum_{i=1}^{N} |\phi_i|^2 - r\right)^2$

$\Rightarrow$ vortex inside vortex string

$$S_{\text{VinV}} = 2\pi r = \frac{(2\pi)^2}{e^2} = \frac{4\pi^2}{e^2} = S_{\text{instanton}}$$

Go back to action in $d = \overset{4+0}{\cancel{3+1}}$

$$\omega = x^3 + i x^4$$
$$z = x^1 + i x^2$$

$$S = \int d^4x \, \frac{1}{4e^2} T_2(F^2) + \sum_{i=1}^{N} |D_\mu q_i|^2 + \frac{e^2}{2} T_r \left(\sum q_i q_i^\dagger - v^2 \right)^2$$

$$= \int d^4x \, \frac{1}{2e^2} T_2 \left(F_{12} - F_{34} - e^2 \left(\sum_i q_i q_i^\dagger - v^2 \right) \right)^2 + |D_z q_i|^2 + |D_w q_i|^2$$

$$+ \frac{T_2}{2e^2} (F_{14} - F_{23})^2 + \frac{1}{2e^2} T_2 (F_{13} - F_{24})^2$$

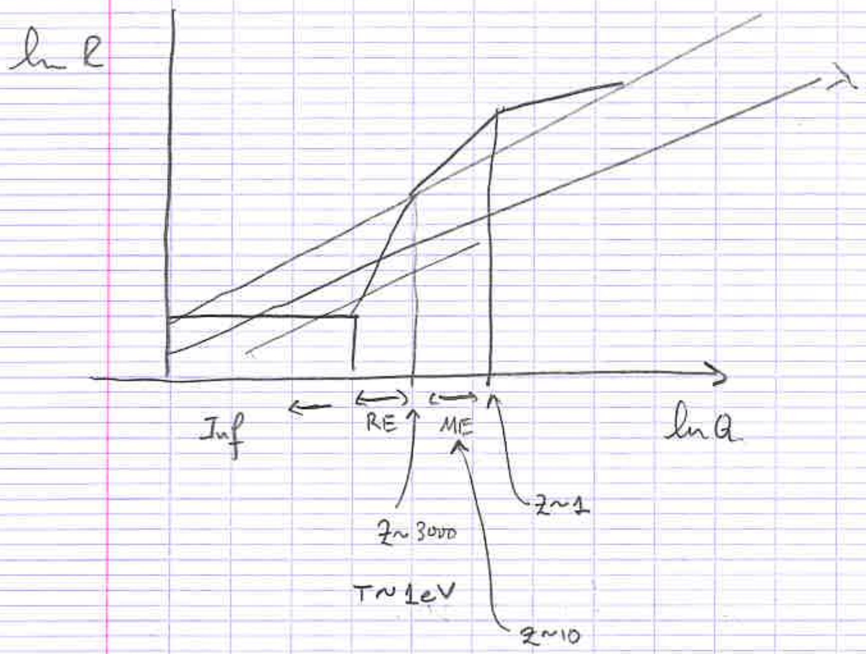
$$+ \frac{1}{2e^2} T_r \, F^* F + F_{12} v^2 - F_{34} v^2 \Big]$$

$$F_{12} - F_{34} = e^2 \left(\sum_i q_i q_i^\dagger - v^2 \right)$$

$$D_z q_i = 0 = D_w q_i$$

$$F_{14} = F_{23} \qquad F_{13} = -F_{24}$$

Zaldarragia III



$$\delta_m = \frac{\delta\rho}{\rho}$$

$$\nabla^2\phi = 4\pi G\rho\,\delta$$
$$= \frac{3}{2} H^2 \delta$$

$\delta_m = \ln t$ RE

$\propto a(t)$ ME

Φ decays RE

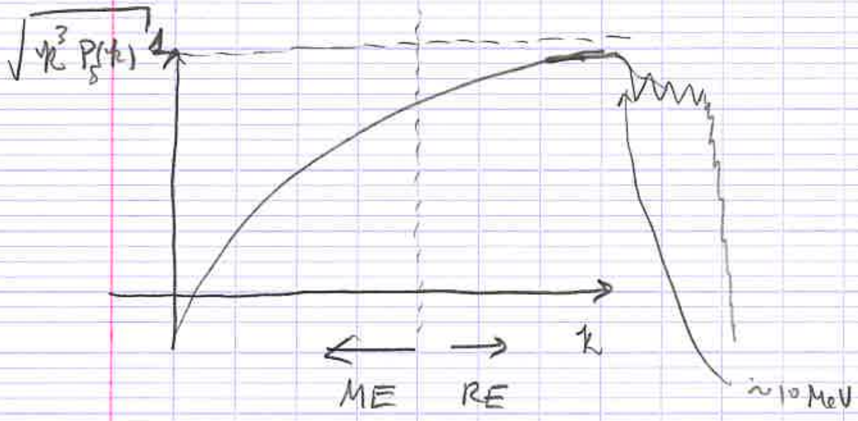
const ME

$\Phi \sim 10^{-5}$

$\Phi \sim \delta$ horizon crossing

$v \sim 200$ km/s galaxy $\Phi \sim v^2$

$v \sim 1000$ km/s cluster of galaxies



λ bigger by factor of 10 $\Rightarrow$ no structure formation

---

M

$\sigma \sim \dfrac{1}{M^2_{\text{cross section}}}$ $\qquad n_\chi \approx e^{-M/T}$

Assume it happen near $T = M$

$$n_\chi^{\text{FREEZE}} \underset{\overset{\uparrow}{M_c^{-2}}}{\langle \sigma v \rangle} \simeq H$$

$$\sim \frac{T^2}{M_{pl}} \sim \frac{M^2}{M_{pl}}$$

$$\rho_\chi(T) = n_x^{\text{Freeze}} M \times (T/M)^3 = \frac{\cancel{M^2}}{M_{pl}} M_c^2 \cancel{M} \frac{T^3}{\cancel{M^3}}$$

$$\rho_\gamma(T) = T^4$$

$$\rho_\chi(T_{eq}) = \rho_\gamma(T_{eq}) \Rightarrow \boxed{T_{eq} = \frac{M_c^2}{M_{pl}^2}}$$

---

for 100 GeV ~ 10 MeV DM moved w/ plasma.

6/24/05

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4) Domain Walls

U(N) gauge theory $\qquad N_f \geq N_c$

$$S = \int d^4x \; Tr\left[\frac{1}{4e^2} F_{\mu\nu}F^{\mu\nu} + \frac{1}{2e^2}(\mathcal{D}_\mu \phi)^2\right] + \sum_{i=1}^{N_f} |\mathcal{D}_\mu q_i|^2$$

$$+ \sum_{i=1}^{N_f} q_i^\dagger (\phi - m_i)^2 q_i + \frac{e^2}{2} Tr\left(\sum_{i=1}^{N_f} q_i q_i^\dagger - v^2\right)^2$$

Lets choose $m_i < m_{i+1}$ (almost WLOG)

Vacua: Each vacuum is determined by a set of N_c distinct elements from N_f

$$\Xi = \left\{ \xi(a) : \xi(a) \neq \xi(b) \text{ for } a \neq b \right\}$$

with $a = 1 \cdots N_c$ $\quad \xi(a) = 1, \ldots, N_f$

$\phi = \text{diag}(m_{\xi(1)}, \ldots, m_{\xi(N_c)})$

which allows me to turn on $q_i^a \sim \delta^a_{i=\xi(a)}$

set $q_i^a = v\, \delta^a_{i=\xi(a)}$

Upshot: # of isolated vacua

$$N_{vac} = \binom{N_f}{N_c} = \frac{N_f!}{N_c!\,(N_f - N_c)!}$$

Symmetry $\quad SU(N_f)_{flavor} \longrightarrow U(1)^{N_f - 1}_{flavors}$ by masses

$U(N_c)_{gauge} \longrightarrow \phi$ by v^2

7 Domain walls

"Bogomolnyi equations" are <u>Domain Wall equations</u>

$$\boxed{\begin{matrix}\partial_0 = \partial_1 = \partial_2 = 0 \\ A_0 = A_1 = A_2 = 0\end{matrix}}$$

$$T_{WALL} = \int d^3_x \Big[\frac{1}{2e^2} \Big[(D_3 \phi) - e^2 \Big(\sum_{i=1}^{N_f} q_i q_i^\dagger \; v^2 \mathbb{1} \Big) \Big]^2$$

$$+ D_3 \phi \cdot \Big(\sum_{i=1}^{N_f} q_i q_i^\dagger - v^2 \Big) \Big]$$

$$+ \sum_{i=1}^{N_f}{}' \left| D_3 q_i - (\phi - m_i) q_i \right|^2 + q_i^\dagger (\phi - m_i) D_3 q$$

$$+ D_3 q_i^\dagger (\phi - m_i) q_i$$

$$\geq - v^2 \int d^3x \, \partial_3 \operatorname{Tr} \phi = - v^2 \Big[\operatorname{Tr} \phi \Big]_{-\infty}^{\infty}$$

⇒ Domain Wall equations

$$D_3 \phi = e^2 \Big(\sum_{i=1}^{N_f} q_i q_i^\dagger - v^2 \Big)$$

$$D_3 q_i = (\phi - m_i) q_i$$

and
$$T_{wall} = v^2 \Big(\sum_{i \in \Xi_-}{}' m_i - \sum_{i \in \Xi_+}{}' m_i \Big)$$

Example: $U(1)$ w/2 flavors $\Rightarrow$ 2 vacua

$$\partial_3 \phi = e^2 \left(|q_1|^2 + |q_2|^2 + v^2 \right) \qquad (e^2 \to \infty \text{ solvable})$$

$$\mathcal{D}_3 \phi = (\phi - m_i) q_i$$

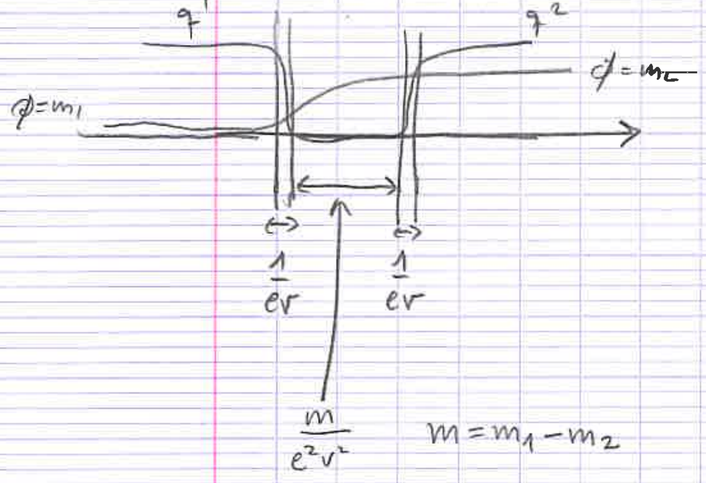
$U(1)_G : q_i \to e^{i\alpha} q_i$

$\phi \to m_1$ at $x \to -\infty$

$U(1)_F : q_1 \to e^{i\beta} q_1$

$\phi \to m_2$ at $x \to +\infty$

$q_2 \to e^{-i\beta} q_2$



$m = m_1 - m_2$

$m \gg ev$

A simple classification of the topological sectors

define N_f-vector $\vec{m} = (m_1, \dots, m_{N_f})$

and write $T_{WALL} = v^2 \vec{m} \cdot \vec{g}$ and write $\tilde{g} = \sum_{i=1}^{N_f - 1} n_i \vec{\alpha}_i$

$\vec{\alpha}_1 = (1, 1, 0, \dots)$

$\vec{\alpha}_2 = (0, 1, -1, 0 \dots)$

$\vdots$

$\vec{\alpha}_{N_f - 1}$

Define the moduli space of domain walls $\mathcal{W}_{\vec{g}}$

$$\dim(\mathcal{W}_{\vec{g}}) = 2 \sum_{i=1}^{N_f - 1} n_i$$

- metrics: [illegible]

Examples of this moduli space:

1) $U(1) + 2$ flavors $\vec{g} = \vec{\alpha}_1$

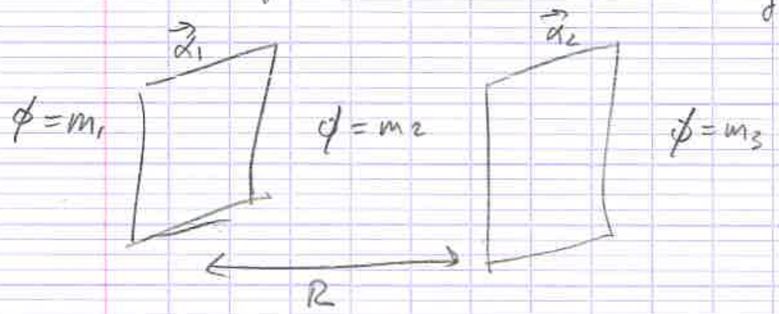
$$\mathcal{W}_{\vec{\alpha}} \cong \mathbb{R} \times S^1$$

phase from flavour symmetry.

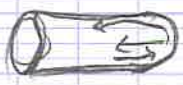
2) 2 domain walls

$U(1) + 3$ flavours

$\vec{g} = \vec{\alpha}_1 + \vec{\alpha}_2$



$$\mathcal{W}_{\vec{g}} = \underset{\substack{\uparrow \\ \text{Center} \\ \text{of mass}}}{\mathbb{R}} \times \underset{\substack{\uparrow \\ \text{center} \\ \text{of phase}}}{S^1} \times \hat{\mathcal{W}}$$

 $\cong \hat{\mathcal{W}}$

Witten cigar metric

black hole

CFT on cigar

$$\begin{tikzcd}
\text{3d thy} \arrow[r] \arrow[d, "\text{classical}"] & \text{quantize} \quad \text{low energy action} \arrow[d, "\text{domain walls}"] \\
\text{domain walls} \arrow[r] & \boxed{\substack{\text{Liouville thy.} \\ \text{2D BH.}}}
\end{tikzcd}$$

$\mathcal{L}_{\text{WALL}}$

Brane Construction for the domain Walls

$$\mathcal{W}_{\vec{g}} \cong \mathcal{M}_{\vec{g}}\Big|_{h^a=0}$$

action on <u>monopole</u> moduli space in plane

Why are domain Walls related to monopoles:

What became of...

<u>Vortices</u>:

Orientational modes come from $SU(N)_{diag}$. But now we only have $U(1)^{N_f-1}$ $\Longrightarrow$ We expect these orientational modes to be lifted.

Vortices that survive are solutions for which

$$V = \mathrm{Tr}\,\frac{1}{2e^2}(\mathcal{D}_\mu\phi)^2 + \sum_{i=1}^{N_f} q_i^\dagger(\phi - m_i)^2 q_i$$

$V=0$ Let's choose $N_f = N_c$ for now

$\phi = \mathrm{diag}(m_1, \ldots, m_{N_f})$ in vacuum

$$\Longrightarrow \quad q^i_a \sim \delta^i_a$$

$\Longrightarrow$ only vortices that survive have

$$B_3 = \begin{pmatrix} 0 & & & \\ & \ddots & & \\ & & B_* & \\ & & & \ddots \end{pmatrix} \qquad q = \begin{pmatrix} v & & & \\ & \ddots & & \\ & & q_* & \\ & & & \ddots \\ & & & & v \end{pmatrix}$$

No masses: vortex moduli space $\mathbb{C} \times \mathbb{CP}^{N-1}$ (for $k=1$)

Add masses: N_c different vortex strings

How do we see this from vortex worldvolume theory?

$U(K)$ + adjoint chiral

+ N_c fundamental massive chiral multiplets

+ FI $r = \frac{2\pi}{e^2}$

Example:

1 vortex in $U(N)$

Theory on the vortex is $U(1)$ + N massive chirals

$$V = \sum_{i=1}^{N} |\psi_i|^2 (\sigma - m_i)^2 + \left(\sum_{i=1}^{N} |\psi_i|^2 - r\right)^2$$

$\Rightarrow$ N isolated vacua

$\sigma = m_i$

$|\psi_j|^2 = r\,\delta_{ij}$

Monopoles:

There are kinks (or domain walls) on the vortex

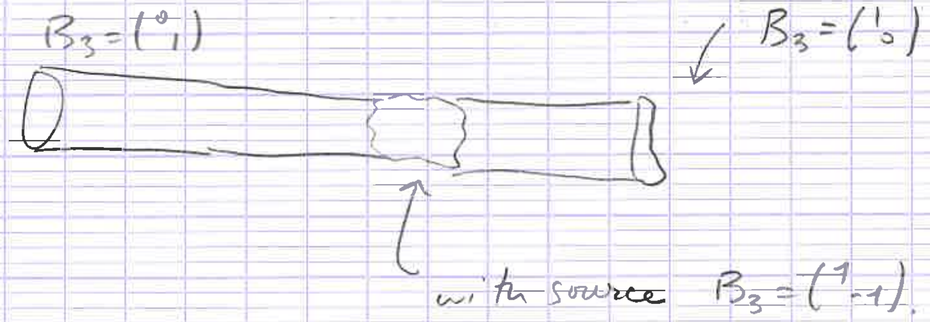
$$\partial_3 \sigma = g^2 \left(\sum_{i=1}^{N} |\psi_i|^2 - r\right)$$

$$D_3 \psi_i = (\sigma - m_i)\,\psi_i$$

$$M_{\text{kink}} = r(\vec{m}\cdot\vec{q}) = \frac{2\pi}{e^2}\vec{\phi}\cdot\vec{g} = M_{\text{monopole}}$$

~~wh~~ do other quantum numbers match

Consider $N_c = 2$



⇒ its a 't Hooft Polyakov monopole.

Go back to 3+1 theory:

$$\mathcal{H} = \int d^3x \, \frac{1}{2e^2}\left(B_3^2 + \underset{\mu=1,2,3}{D_\mu}\phi^2\right) + \sum_{i=1}^{N} |D_\mu q_i|^2$$

$$+ \sum_{i=1}^{N} q_i^\dagger (\phi - m_i)^2 q_i + \frac{e^2}{2} \operatorname{Tr}\left(\sum q_i q_i^\dagger - v^2 \mathbb{1}\right)^2$$

$$= \int d^3x \, \frac{1}{2e^2}\left[(D_1\phi + B_1)^2 + (D_2\phi + B_2)^2\right.$$

$$+ \left(D_3\phi + B_3 - e^2\left(\sum_i q_i q_i^\dagger - v^2\right)\right)^2$$

$$+ \sum_i |D_1 q_i - iD_2 q_i|^2 + \sum_{i=1}^{N} |D_3 q_i - (\phi - m_3) q_i|^2$$

$+ \cdots$ where $\cdots$ $T_{\text{mono}} + T_{\text{wall}} + T_{\text{vortex}}$

New equations

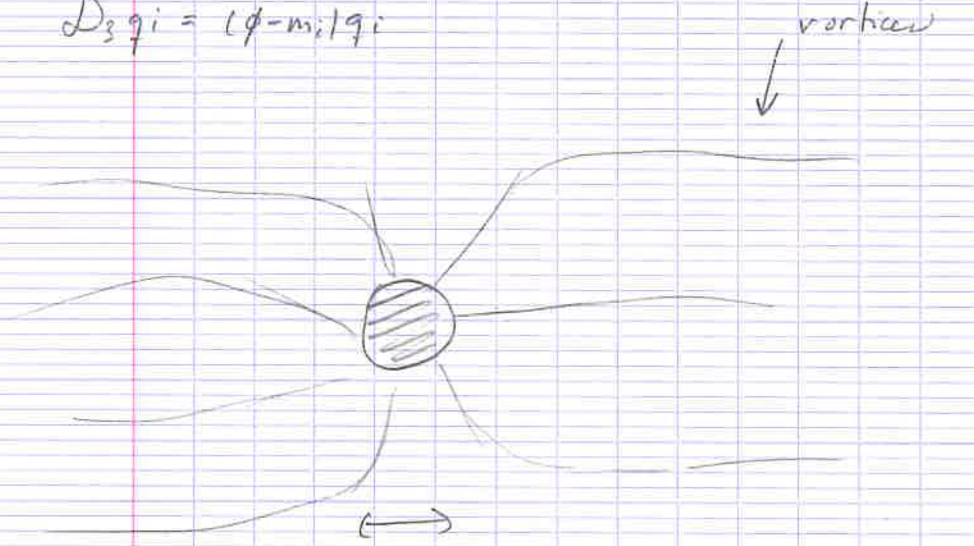
$$D_1\phi + B_1 = 0$$

$$D_2\phi + B_2 = 0$$

$$D_3\phi + B_3 - e^2\left(\Sigma q_i q_i^\dagger - v^2\right) = 0$$

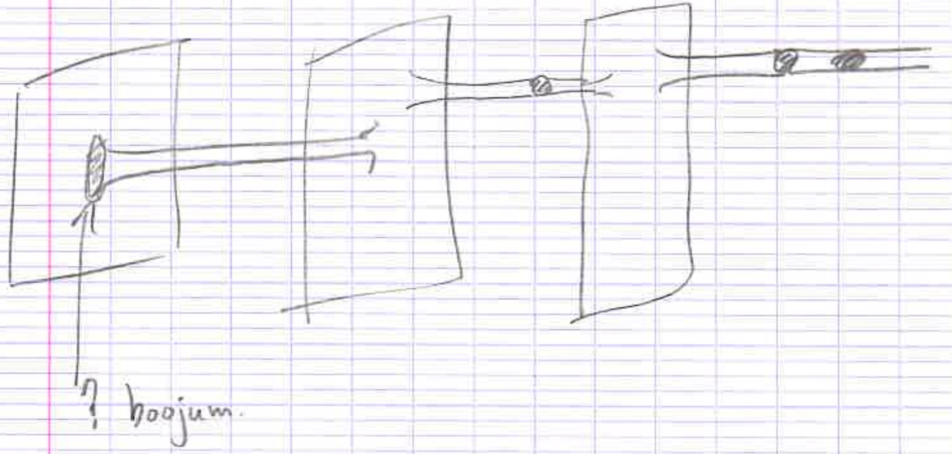
$$D_1 q_i = i D_2 q_i$$

$$D_3 q_i = (\phi - m_i) q_i$$



$\frac{1}{m} \sim \frac{1}{\phi}$

Hanani - Witten



Punch line:

4D Theory $U(N) + N_f$ flavours, masses m_i
coupling e^2

2D theory (on vortex string). $U(1)$ thy +
N massive charged chiral multiplets.
FI $r = \frac{2\pi}{e^2}$ $\mathcal{N}=(2,2)$

$M_{\text{kink}} = M_{\text{monopole}}$ survives in full quantum theory

$$M = M_{\text{classical}} + M_{1\text{-loop}} + \sum_{n=1}^{\infty} M_{n\,\text{instanton}}$$

$\boxed{\text{2D } \sigma \text{ model calculate 4D stuff}}$

$r = \frac{2\pi}{e^2}$ $\qquad r(\mu) = r_0 - \frac{N}{2\pi}\log\left(\frac{\mu_{UV}}{\mu}\right)$

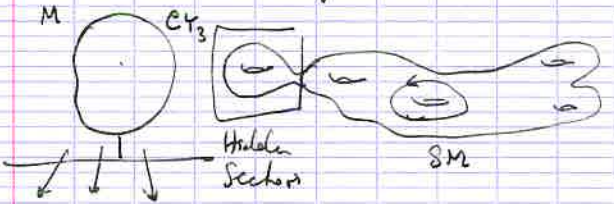
6/27/05

Douglas I

0 - What is a vacuum $U \leftarrow V'_{eff}=0 \quad V''=0$

1 - Is the # of vacua of vacua finite.

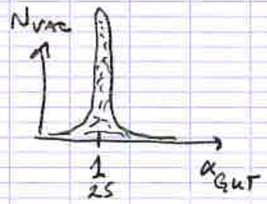
potentially realistic



#of UV completion of SM? finite?

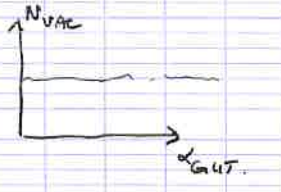
2- Are distributions peaked on uniform?

- peaked



(central limit thm.).

- uniform



(but, uniform is what?!)

- natural

M- theory Freund-Rubin compactification $AdS_4 \times S^7$

$$R_{ij} - \frac{1}{2} g_{ij} R = T_{ij} = (\sigma, \sigma); \quad N = \frac{1}{\cdots} \int_{S^7} * \sigma^4 \quad \text{M-theory}$$

$N \in \mathbb{Z}$

near horizon limit of N M2 branes

$$\left(1-\frac{N}{r^6}\right)^{1/3} dx_{\parallel}^2 + \left(1-\frac{N}{r^6}\right)^{1/3} (dr^2 + r^2 d\Omega^2)$$

$R_{\substack{AdS \\ S^7}} = N^{1/6}$ ~~A~~ $\Lambda_{AdS} = -\frac{1}{R^4}$

$V_{S^7} \sim N^{7/6}$

∞ # of vacua.

but...

$\boxed{M^2_{pe\,4}} = V_M M^9_{pe\,11}$ If V_M is too big $M^9_{pl\,11}$ gets too small, ~~to~~ and Q.G. effects would appear in experiments.

$d>4$

$V_M < V_{MAX} \Leftarrow$ observable M fundamental $\gtrsim 10\,\text{TeV}$ (?).

SUSY

$\text{diameter}_M < d_{max} = \max_{x,y} d(x,y)$

$$\Delta \varphi_i = m_i^2 \varphi_i^2 \qquad \underbrace{m^2_{KK} \le \frac{1}{d^2}}_{m_{KK} = m_i \text{ for some } i.}$$

Kaluza Klein scale.

What about $AdS_4 \times S^7/\mathbb{Z}_K \Rightarrow V_{S_7} = N^{7/6}/K$

$$\to N' = \int_{S^7/\mathbb{Z}_K} * \sigma^{(4)} \in \mathbb{Z} \Rightarrow N = KN'$$

$$\Rightarrow \cancel{\Delta}_{pe} \quad V_{S_7} \approx (KN)^{7/6}/K$$

Non geometric versions of above conditions?

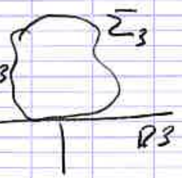
<u>Attractor problem:</u> in IIB CY_3 $N=2$, $\alpha=4$

$b^3 \to$ vectors complx. str. $\int C^{(4)}$

$b^{1,1}$ hyper

$\frac{1}{2}$ BPS state

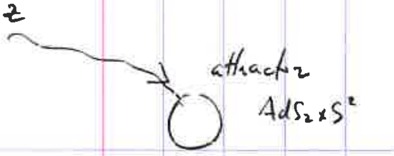
charge $Q \in H^3\ CY^3$



central charge

$$Z = \int_{\Sigma^3} \Omega^{(3)}_{z} \qquad \Pi^A = \int_{\Sigma A} \Omega$$

$$= Q_i\, \overline{\Pi}^A(\hat{z})^*$$



$$\partial_r \varphi = -g^{ij}\,\partial_j |Z|^2$$

$S \sim |Z_*|^2$

fixed pt $\partial_j \|Z\|^2 = 0$

$$\begin{array}{c} \uparrow \\ \dfrac{|Z|^2}{\int_M \Omega\wedge\Omega} \end{array}$$

special geometry

Aganagic I:

Topological Strings and Dualities

T.S on CY manifolds

CY d-manifold X, Riemannian w/ g

1) X is complx. d-manifold, patches $\mathbb{C}^d = \{z_1, \dots, z_d\}$ with holomorphic transition fns.

g is hermitean $\quad g_{ij} = g_{\bar i \bar j} = 0 \qquad g_{i\bar j} = (g_{\bar i j})^*$

define Kahler form ~~k~~ $k = g_{i\bar j}\, dz^i \wedge dz^{\bar j}$

2) X is Kahler $\quad dk = 0 \qquad k \in H^2(X, \mathbb{R})$.

3) X is Ricci flat $\quad R_{i\bar j} = 0 \quad \Rightarrow$ CY

Yau thm:

If X complex Kahler, Ricci flat metric exists iff $c_1(TX) = 0 \Rightarrow \exists$ unique, nowhere vanishing holomorphic form $\Omega^{d,0}$

$\updownarrow$ up to rescaling

[equiv. to choice of cmplx. structure].

String thy. in X:

$N=1$ SUSY sigma model on X. $\qquad \phi : \underset{\text{Riemann}}{\Sigma} \to X$

$$S = \int_\Sigma d^2\sigma \; g_{i\bar j}\, \partial\phi^i \partial\phi^{\bar j} + \psi_-^{\bar i} D_+ \psi_-^j + \psi_+^{\bar i} D_- \psi_+^j$$

$$+ R_{i\bar j k\bar e}\, \psi_-^i \psi_-^{\bar j} \psi_+^k \psi_+^{\bar e}$$

$$\psi_\pm \in \Gamma(S_\pm, TX)$$

$\uparrow$ section of

- When X is Kahler σ-model has $N=2$ SUSY

- σ-model is quantum conformal at 1-loop
  when X is Ricci flat $R_{i\bar{j}}=0$

But; unless Σ is flat SUSY is broken
(No covariantly constant sections) $\nexists\ D_\pm \varepsilon = 0$

Way around this problem: Topological <u>twisting</u>
$\downarrow$
Changing spin assignments
⇒ scalar SUSY

<u>$N=2$ superconformal invariance SCFT:</u>

Algebra: $T(z)$ stress tensor spin 2
$G^\pm$ super-current
$J(z)$ $U(1)$ current (charge of J → left fermion #)

$$T(z) \to T(z) - \frac{1}{2} J(z)$$

$$(h, q) \to (h - \tfrac{1}{2}q, q) \qquad D\varepsilon = 0 \to \partial\varepsilon = 0$$

⟺ turn on a background $U(1)$ connection = spin connection

<u>Right twist:</u>

$$T(\bar{z}) \to \bar{T}(\bar{z}) \mp \frac{1}{2}\bar{\partial}(\bar{z})$$

A-type $\overset{L\ \ R}{(-,-)}$
B-type $(-,+)$ (1,1) SUSY

After twisting

- from 2 SUSY's: topological BRST type charge Q. <u>Nontrivial observables</u>: ~~the~~ cohomology $QO=0$ $\quad O \sim O + Q\tilde{O}$

  $T = Q \cdots \}$ ⇒ correlations functions indep of worldsheet metric.

- Original σ-model depended on both $\underline{\underline{k}}$ & $\underline{\underline{\Omega}}$

  A-type depends only on choice of $\underline{\underline{k}}$

  B-type " " " " " $\underline{\underline{\Omega}}$

- Because of SUSY, Path Integral will localize near configurations preserving SUSY.

In A-type <u>Hol. maps</u> $\quad \phi : \Sigma \to \underline{X} \quad \bar{\partial}\phi = 0$

Genus zero + counting of rational curves

In B-type $\partial\phi = 0 = \bar{\partial}\phi$

why? - Indep. of $\underline{\underline{k}}$ $\quad Vol(\underline{X}) = k\wedge\cdots\wedge k$

⇒ B-type string is a <u>point particle theory</u>.

- Genus zero amplitudes in B-type can be computed from <u>classical Geometry</u>

To get Type A, Type B string, sum over all Riemann surfaces

Introducing D-branes:

Topological strings on Riemann surfaces w/boundaries

Boundary conditions must preserve SUSY : or topological invariance

A type:

Boundaries on Lagrangian submanifold L

$k|_L = 0 \Rightarrow \dim_{\mathbb{R}} L = d$. + Flat bundle

B model: Hol submanifolds of X with hol bundle $F_{2,0} = 0$ $F = dA$.

$\exists$ string theory for all dim CY's. But in $D_{\mathbb{C}} = 3$ it is most interesting (in other dimension most of amplitudes (Σ_g $g > 0, 1$) vanish.

Why topological strings?

- A(B) - type topological strings compute F-type terms (Integrals over ½ superspace) in comp. of IIA(IIB) string on CY_3-fold with $N = 1, 2$ SUSY
- Laboratories for dualities. Simple laboratories ~~for~~ where superstring dualities can be studied.

    - Open - Closed

B-type topological String

- Calabi-Yau manifolds:

$\underline{X}$: noncompact CY

$yz = H(x,p)$ $x, y, z, p \in \mathbb{C}$. H a polynomial.

Ex. $H(x,P) = p^2 + \prod_{i=1}^{m} (x - a^i)$ $a^i \in \mathbb{C}$ parameters

hol. 3-form $\Omega^{3,0} = \frac{1}{2}\, dz \wedge dp \wedge dx$

a_i's parametrize choices of cmplx structure on $\underline{X}$.

B-model amplitudes → depend on a^i, $g=0$ amplitude can be computed from classical geometry.

How?

$$F_{g=0} = \int D\phi\, D\psi\, \langle \dots \rangle\, e^{iS}$$

$$\Omega^{3,0} \in H^3(\underline{X}, \mathbb{C})$$

periods: $\oint_{\gamma^e} \Omega$, $\gamma^e \in H_3(\underline{X}, \mathbb{Z})$

$\underline{X}$ $A^I, B_J \in H_3(\underline{X}, J)$ $\#(A^I, B_J) = \delta^I{}_J$

$\#(A^I, A^J) = \#(B^I, B^J) = 0$

$I, J = 0, \dots, \underbrace{\dim(H^{2,1})}$

↑ dim. of cplx. str. moduli space.

$t^I = \oint_{A^I} \Omega$ (one too many A^I's) since

$\Omega \sim \lambda \Omega$

$t^I \sim \lambda t^J$ } projective coordinates.

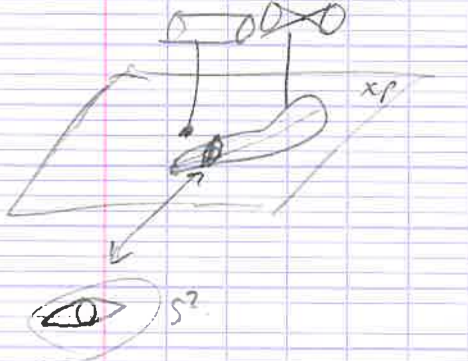
$F_J = \oint_{B_J} \Omega$ $\quad F_J = F_J(t)$ $\quad \partial_I F_J = \partial_J F_I$

$$F_J = \frac{\partial}{\partial t^J} F_0(t)$$

this is the genus zero amplitude we are after.

$\boxed{F_0}$ = genus zero amplitude.

X : $yz = H(x,p)$ $\quad \Omega = \frac{1}{2}\, dz \wedge dp \wedge dx$



X is a fibration by cyllinders $yz = 1$ over xp plane.

$yz = 0$

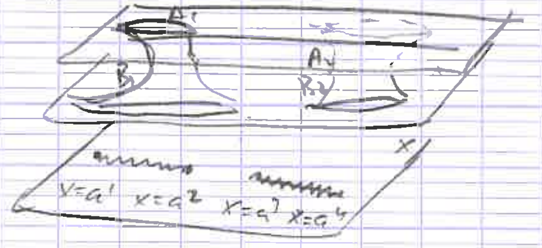
$y = 0 \quad z_0 = 0$

$\Sigma_c : H(p,x) = 0$

A : B cycles on X decend to 1 cycles on Σ

Ex. $H(x,p) = p^2 + \prod (x - a^i)$

Σ



$$p = \pm \sqrt{\prod (x - a^i)}$$

$$\int_{A_i(B_i)} \Omega = \int \frac{dz}{z}\, dp \wedge dx = \int_{\hat{A}(\hat{B})} p\, dx$$

Bousso I

Holography, Cosmology & Observables in Quantum Gravity

I. Covariant Entropy Bound

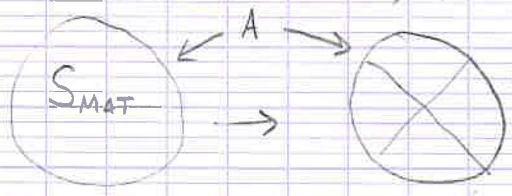
coarser entropy bound

Is BH formation and evaporation a unitary process.

Area theorem: $dA \geq 0$

$A = 4\pi R_H^2$

No hair theorem: M, Q, J



← no matter fields here

$S=0$ $\qquad$ $S = \ln(\#\,\text{states})$

$$S = \frac{Ac^3}{G\hbar} \; ; \; G\hbar = l_p^2$$

$$\boxed{S_{MAT} \leq \frac{A}{4G\hbar}}$$

1) $S \not\sim V$

2) Not dependent on matter theory

Can this be right?

Imagine breaking $S \leq \frac{A}{4}$

$S \sim VT^3 = R^3T^3$ violates $S \leq \frac{A}{4}$ for R large enough

but include gravity

$E < \frac{R}{2}$ (or else it collapses)

$$E \sim VT^4 \Rightarrow T^4 \lesssim \frac{1}{R^2} \Rightarrow S \lesssim R^3 \cdot \left(\frac{1}{R^2}\right)^{3/4} \sim R^{3/2} \sim A^{3/4} \ll A \; !$$

Can't break bound due to gravitational collapse.

Causal Diamond Paradigm :(Complementarity)

$S_{BH} = \frac{A}{4}$; $E = M$ $\quad dS = \frac{dE}{T} \Rightarrow \boxed{T = \frac{1}{4\pi R}}$

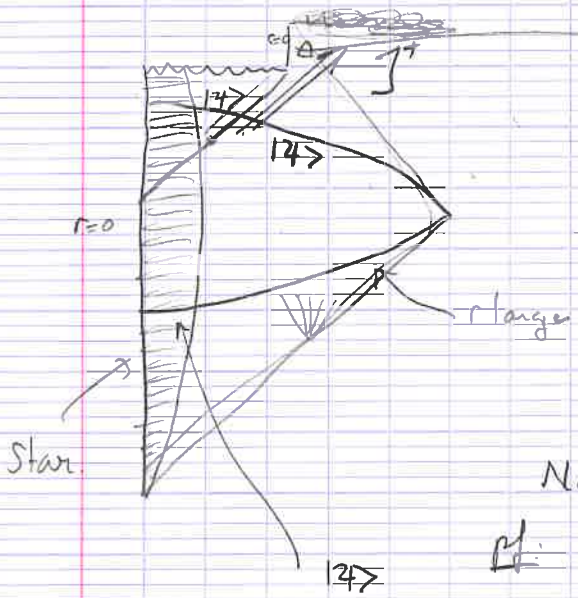
maximally
mixed
quantum
state.

Hawking : Exactly thermal.

$dE = -AT^4 dt$ Boltzmann law

$\sim \frac{-1}{E^2} \Rightarrow \Delta t_{evap} \sim E^3$

QM



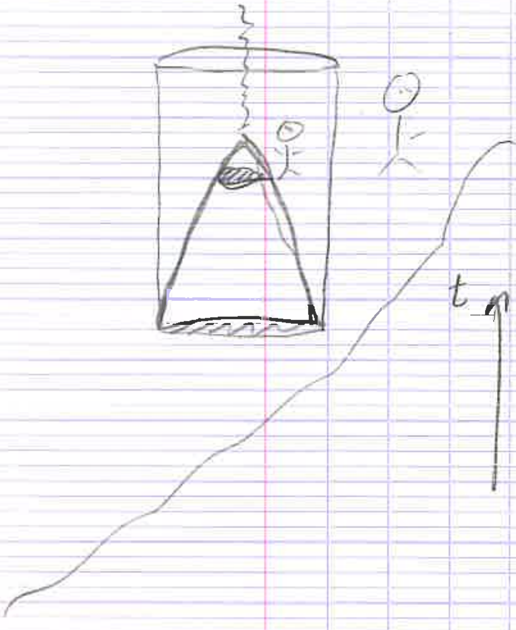
No cloning theorem. $|\psi\rangle \to |\psi\rangle \otimes |\psi\rangle$

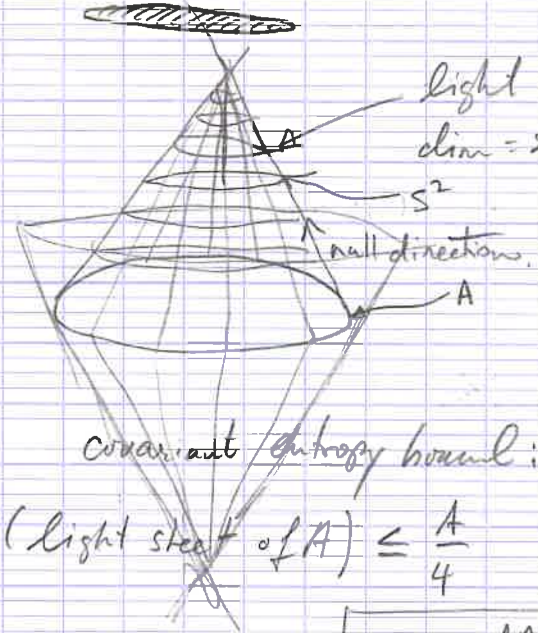
cf. $|\psi\rangle + |\phi\rangle \longmapsto |\psi\rangle|\psi\rangle + |\phi\rangle|\phi\rangle$

$|\psi\rangle + |\phi\rangle \longmapsto |\psi\rangle|\psi\rangle + |\phi\rangle|\phi\rangle$
$+ |\psi\rangle|\phi\rangle + |\phi\rangle|\psi\rangle$

Covariant E. B.

$S \le \frac{A}{4}$ not quite correct.





Covariant entropy bound:

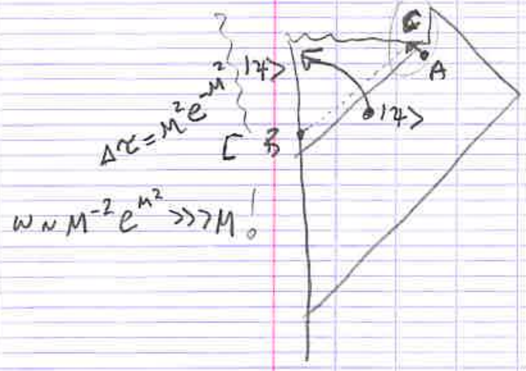
which of the 4?

where to stop?

$$S(\text{light sheet of } A) \le \frac{A}{4}$$

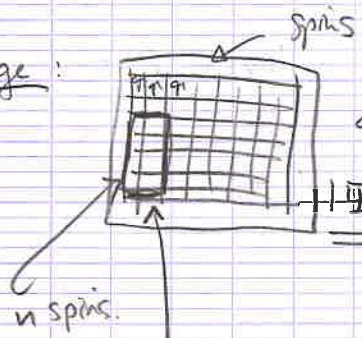
$$\boxed{\theta = \frac{dA/d\lambda}{A} \le 0}$$

Causal Diamond Paradigm.



measure Hawking radiation ? jumps in.

Page:



← typical pure state. (i.e. entangled)

$|\Psi\rangle$ = entire state (extracts max ammounts of inf.).

$|\phi\rangle$ = state of the n spins

n - Trpling

↑

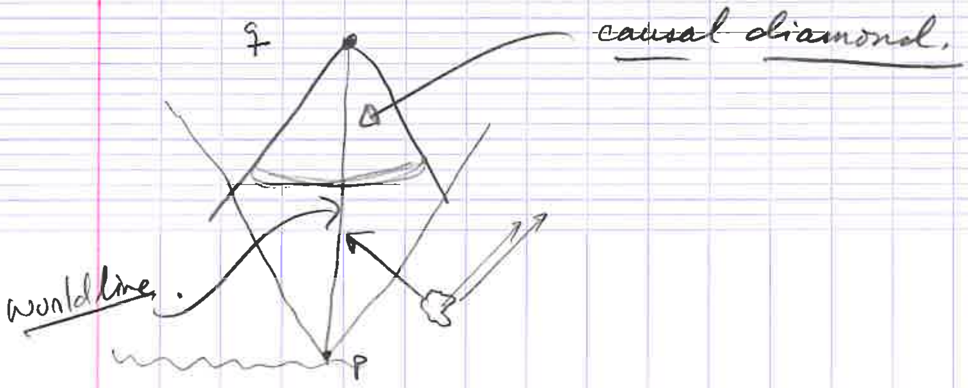
measures purity.

If $|\Psi\rangle = |\phi\rangle \otimes |\delta\rangle$ you get n bits of infor

need to measure at least half to get any info.

⇒ outside observer must wait for $\sim E^3$

Lesson: you can't apply quantum mechanics globally.



Aganagic

D-type D-branes

D1 branes – B-type branes on holomorphic curves

X: $yz = H(p,x)$ $x, y, z, p \in \mathbb{C}^4$ ← $\nexists$ holo. curves.

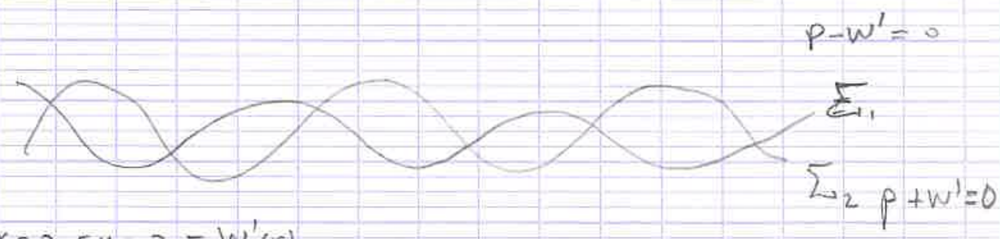
$\downarrow$

X_* $yz = p^2 - (W'(x))^2$ $W'(x)$ = Poly. of order n in x.

$W'(x) = \prod_1^n (x - b^i)$

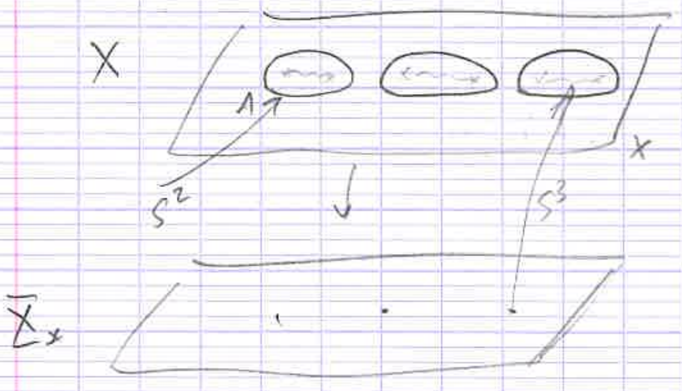
Σ' $H(x,p) = 0$ $p^2 - (W'(x))^2 = 0$

Σ':



$x = 0 = y = z = W'(x)$

→ singularities



2n roots come together in pairs.

$H_* = p^2 - (W')^2$

$H = p^2 - W'^2_{(x)} + P_n(x)$

Another way to get smooth CY

$$X_* \to X_T$$

I. $yw = p - W'(x)$ $\qquad \omega = \frac{1}{w'} \qquad y = (p - w'(x))\frac{1}{w}$

II. $zw' = p + W'(x)$ $\qquad x = x$

$\qquad z = \omega(p + w')$

sing $y = 0 = z = p = W'(x)$ in X_* $\Big\{$ n $\mathbb{P}^1$'s

$p' = \omega_1 w' \qquad \omega = {}'\omega$

Wrap branes on these $\mathbb{P}^1$'s:

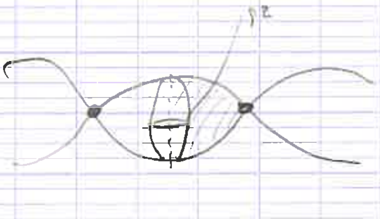
For 1 D1 branes on curve C

Action is $S_{brane} = \int_{B(C,C^*)} \Omega^{3,0}$ $\qquad B(C,C^*)$ connects C to C'

Action of N D5's → Hol. CS Action

$$S = \int_X \Omega \wedge \mathrm{Tr}\left(A\bar{\partial}A - \frac{2}{3}A^3\right) \qquad F = 0$$

Dijkgraaf & Vafa
1st paper on matrix models



$$S = \int_B \Omega =$$

$$= \int \frac{dz}{z} \wedge dp \wedge dx$$

$$= \int p_1(x)\,dx - \int p_2(x)\,dx = W(x) + W(x) = 2W(x)$$

Action is $W(x)$. → minimum $\frac{\delta S}{\delta x} = W'(x) = 0$ $\boxed{x = b^i}$

<u>N</u> D1 branes

~~$S = \frac{1}{g_s}\int$~~ $S = \frac{1}{g_s} \mathrm{Tr}\, W(\bar{X})$ (exact in α').

$\bar{X}$ = N×N hermitian matrix.

$$Z = \int dX\, e^{\frac{\mathrm{Tr}(W)}{g_s}} \left[\frac{1}{\mathrm{Vol}(U(N))}\right].$$

$$\frac{\delta}{\delta X^{ij}} \mathrm{Tr}\, W(\bar{X}) = 0 . \quad \left(W'(\bar{X})\right)_{ij} = 0$$

$$\bar{X} = \begin{pmatrix} b_1 & & & & & & \\ & \ddots & & & & & \\ & & b_1 & & & & \\ & & & b_2 & & & \\ & & & & b_2 & & \\ & & & & & b_n & \\ & & & & & & b_n \end{pmatrix} \begin{matrix} \}\, N_1 \\ \}\, N_2 \\ N_n \end{matrix} \qquad \sum_i N_i = N.$$

Evaluate P.T.

$\bar{X}_{i\bar{j}}$ double line. Ribbon graphs.

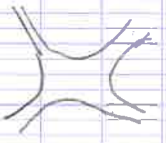
$\frac{1}{\lambda}\mathrm{Tr}\, X^2$

$\sim \lambda$

$\frac{1}{\lambda}\mathrm{Tr}\, X^3$

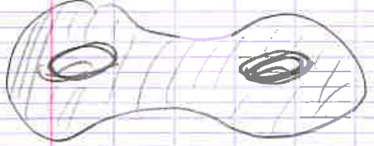
$\sim \frac{1}{\lambda}$

$\frac{1}{\lambda}\mathrm{Tr}\, X^4$



$\sim \frac{1}{\lambda^2}$

$\langle T_2 X^2 \, T_2 X^3 \rangle \quad , \quad \langle T_2 X^4 \rangle$



Open string diagram

Open String diagram

$$Z = e^{F(g_s, N, b_i)} = e^{\sum_{g,h} F_{g,h}(b) \, g_s^{2g-2+h} N^h}$$

moduli

At large N, this theory looks like closed string th.

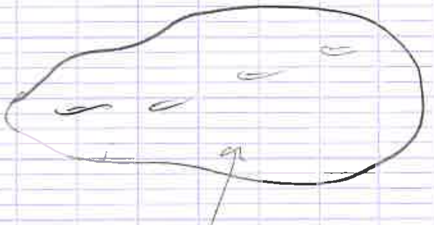
$$F = \sum_{g,h} F_{g,h} \, g_s^{2g-2+h} N^h$$

g_s ~ small

$t = g_s N$ finite

$$= \sum_g \Big(\sum_h F_{g,h} (t)^h \Big) g_s^{2g-2} = \sum_g F_g(t)$$

= closed string expansion.



many holes

What is this closed string theory?

Compute genus ~~zero~~ amplitude:

$\downarrow$

summing planar graphs ($\exists$ methods).

$$X = U \begin{pmatrix} x_1 & & \\ & \ddots & \\ & & x_n \end{pmatrix} U^{-1}$$

$$\int \frac{dX}{\mathrm{Vol}(U(N))} = \int \frac{dU\, d\Lambda}{\mathrm{vol}} J(X) = \int \prod d\Lambda_i \, \Delta(\Lambda)^2$$

$$\Delta(X) = \prod_{i<j} (X_i - X_j)$$

$$Z = \int \prod dx_i \quad \Delta(x_i)^2 \quad e^{\left(\sum_{i=1}^{N} \frac{W(x_i)}{g_s}\right)}$$

at large N: saddle part of effective action

$$\sum \frac{W(x_i)}{g_s} + \sum_{i<j} 2 \log (x_i - x_j)$$

$$W'(x) - 2 g_s \sum_{j \neq i} \frac{1}{(x_i - x_j)} = 0$$

(effective action of 1 e'val)

$$\boxed{p(x) = W'(x) - 2 g_s \mathrm{Tr} \left(\frac{1}{x - X} \right)} = \frac{\delta}{\delta x} S$$

compute large N expectation value of this

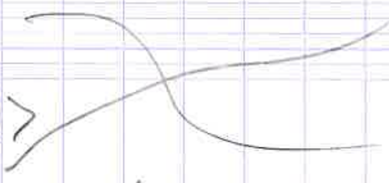
Interpretation: effective action of 1 eval.

A D1 brane on Σ

$$\int_{B(C, C^*)} \Omega = \int_{x^*}^{x} p\, dx$$

1 form on Σ.

$$p = W'(x) + \left\langle \mathrm{Tr} \left(\frac{1}{x - X} \right) \right\rangle$$



$$p(x) = W'(x) + \left\langle \mathrm{Tr} \left(\frac{1}{\cancel{x}x - X} \right) \right\rangle$$

$\updownarrow$

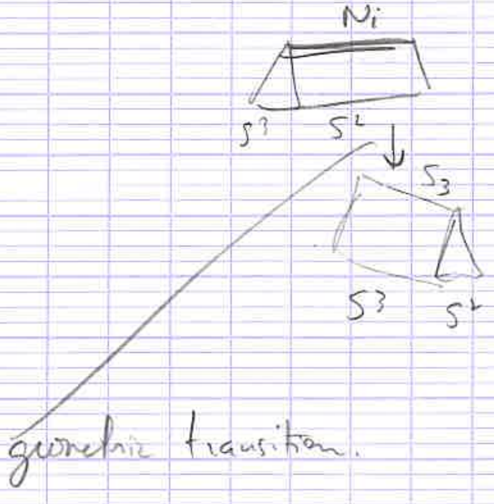
Branes deform the geometry.

$$\boxed{p^2(x) - (w'(x))^2 + \underset{\uparrow \atop \text{poly of order } n.}{f_n(x)} = 0} \quad \Leftarrow \text{answer.}$$

$$\int_{A_1} \rho\, dx = N_i g_s = t_i$$

Geometry near $\mathbb{P}^1$'s (resolved conifold)

for $N \rightarrow$ large closed string thy.



$$\int_{S^3} \Omega = N_i g_s$$

D-branes are sources of charge whose flux is measured by Ω.

$$d\Omega = \underset{\uparrow \atop \#\text{ branes}}{N} g_s\, \delta(\underset{\uparrow \atop \text{curve where brane lives}}{C})$$

$$\oint_{\substack{3 \text{ cycle} \\ \text{linking}}} \Omega = \underset{\uparrow \atop \#\text{ branes on } C}{N g_s} \qquad C$$

D1 branes have a privilidged rôle, as sources of gravity

Back to Closed B-model theory

The SFT is just a field theory Kodiara-Spencer theory of gravity

"Quantize variations of complex structure"

Bershatski, Ooguri, Vafa.

$yz = H(x,p)$. ← variations of complex structure preserving $H = yz$

$\Sigma : H(x,p) = 0$

→ 6D theory goes 2D

KS → 2D theory on Σ_1'

meromorphic nonzero form on Σ_1'

Ω on X → 1 form λ on Σ $\lambda = p\,dx$

$p = p(x)$ solves $H(p,x) = 0$.

allow changes in λ. ↔ allow complx structure to vary.

$$p(x) \to p + \delta p.$$

$\delta\lambda = \delta p\, dX$

$\bar{\partial}\delta p = 0$ A.E.

δp

$\delta\Lambda = \partial\varphi(x)$ $\varphi(x)$ chiral scalar field $\partial\bar\partial\varphi$ $\varphi \to \varphi + \bar\partial\rho$

$\Rightarrow$ parametrize variations ~~of~~ by free chiral scalar field.

But, $\lambda = p\,dx + \partial\varphi_1 = \partial\varphi(x)$ $\varphi = \varphi_{cl} + \varphi_{qu}$

$$p\,dx = d(px) - \partial\tilde\varphi(p)$$

Legendre Transform $\boxed{\varphi(x) = px - \tilde\varphi(p)}$ ← Quantum Mechanically?

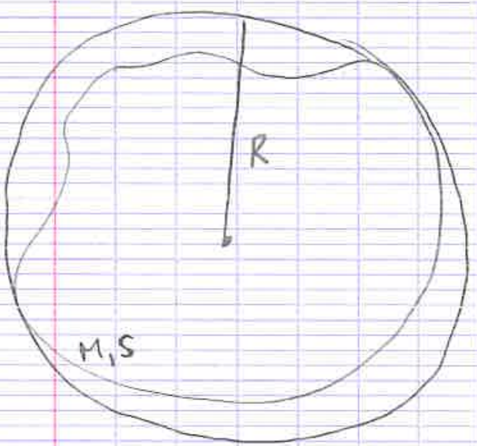
6/28/04

Bousso II

Holography and flat space:

weakly gravitating systems:

Beckenstein bound



$M \ll \frac{R}{G_N}$ "weakly gravitating"

$$S \leq \frac{2\pi MR}{\hbar} \ll \frac{\pi R^2}{G\hbar}$$

electron: $R \sim \frac{\hbar}{M}$

holographic bound $\frac{A}{4G\hbar} \sim \frac{\ell_{Comp}^2}{\ell_{pl}^2} \sim 10^{44}$

Bek bound $\frac{2\pi MR}{\hbar} = O(1)$

actual entropy $O(1)$.

$S_{\text{light sheet}} \leq \frac{A}{4}$

note also $S(V) \leq S(L)$

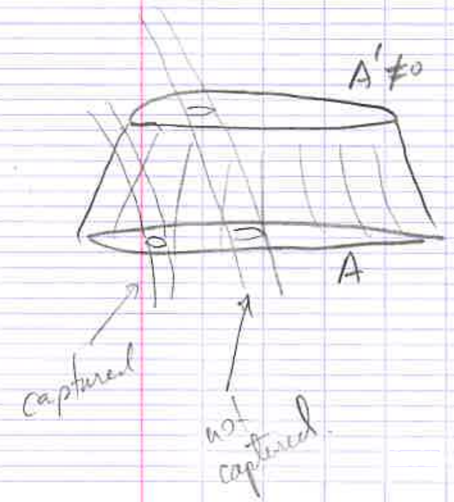
$A_1 > S_{\text{Star}}$

$A_2 > S_{\text{Star}} + \text{Rock}$

GSL for bh formation.

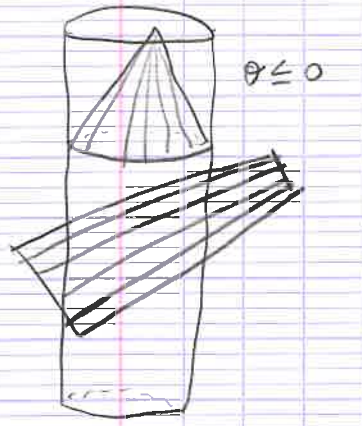
$$\frac{A_2 - A_1}{4} \geq S_{\text{rock}}.$$

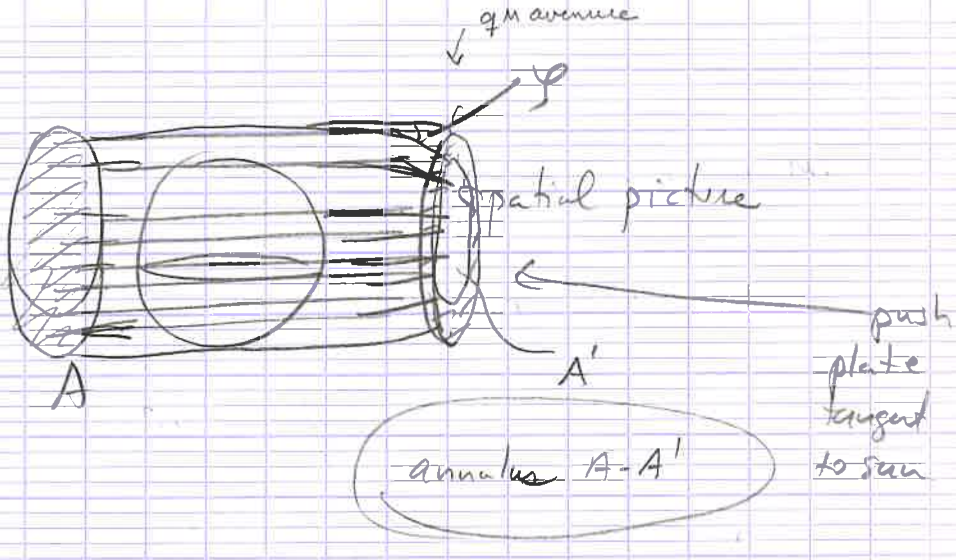
But not GSL for adding.



$$S \leq \frac{A - A'}{4}$$

This version implies Beckenstein's bound





worldvolume

$$\varphi \sim \frac{GM}{R}$$

$$\frac{d\theta}{d\lambda} = \dot{\theta} - \frac{1}{2}\theta^2 - 8\pi G T_{\mu\nu} k^\mu k^\nu$$

[illegible]

width $R\varphi \sim GM$

$\Rightarrow$ $A - A' = GMR$

$\Rightarrow$ $S \lesssim \frac{GMR}{G\hbar} \sim \frac{MR}{\hbar}$ Beckenstein

what if density is inhomogeneous?

CAN fix this. → integrate Raychaudhuri eqn

$\theta(\lambda)$

$A(\lambda)$ ✓

A careful calculation using Ray's eqn

$$\rightarrow \boxed{S \leq \pi M w/\hbar} \leq \frac{2\pi MR}{\hbar}$$

w = length of longest lightray.

What is S on L.H.S.

cost of boundary conditions → box must be extensive.

Complete system and entropy

QM in flat space as consequence of GR:

Deriving uncertainty principle:

- consider $S \leq \frac{A-A'}{4G\hbar}$ $G, \hbar$
- use classical GR: $G_{ab} = 8\pi T_{ab}$

⇒ Bekenstein Bound $S \leq \frac{\pi M w}{\hbar}$

- which could be violated if one could have $\Delta x \Delta p \ll \hbar$

⇒ $\Delta x \Delta p \gtrsim \hbar$

light rays are blind to vac energy.

Relation of Bek. bound to AdS/CFT?

α' corrections | locality

Flannigan, Marolf, Bousso.

Try to write rigorous definition of entropy:

- Specify "b.c." $M, w =$ width
- Count $\# N$ of fock space $\overset{\text{bound}}{\vee}$ states, with

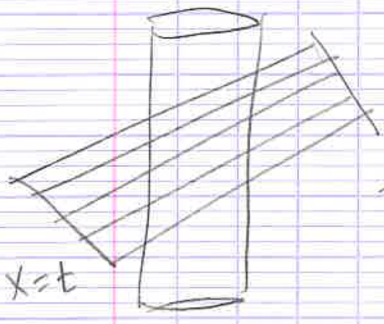
$E \leq M$ & localized to a width w.

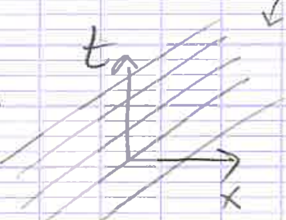
$\rightarrow S = \ln N$

Shortcomings:

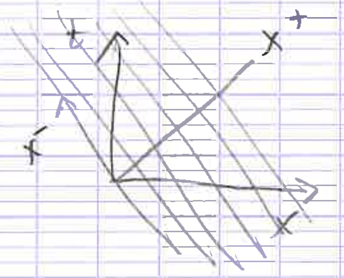
- Say $w = 1 \dot{A}$ (does Hydrogen fit?), where do we cut off wave function?
  → WIDTH AMBIGUITY
- We specify M, w; but should only specify ΔA.
  → Maybe → $M \cdot w$

Intermediate result of derivation:





light-cone foliation.



light front frame.

cf

4mom.

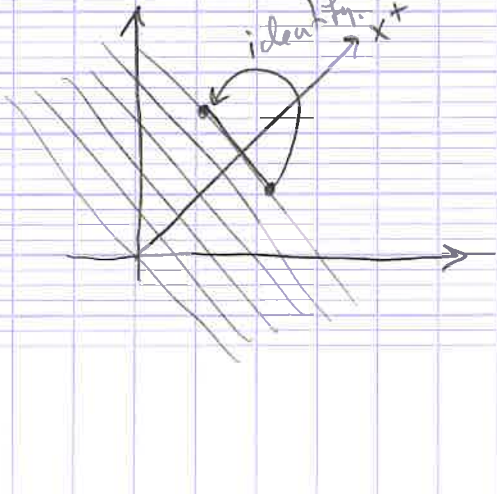
$$S \leq \pi \underbrace{(P_a k^a)}_{\alpha} \underbrace{\Delta\lambda}_{\frac{1}{\alpha}}$$

(in rest frame RHS = Mw

$P_- = P_a k^a$

bound states in QFT → Light front frame

fix $\Delta\lambda$



DLCQ.

$$P_- = \frac{2\pi K}{\Delta\lambda} \hbar$$

$\rightarrow S \leq 2\pi^2 K$

$K = 1$ <u>species problem</u>.

How about in context of Matrix theory

6/28/05

AdS/QCD

J.P, Strassler [redacted]

.0003136
0109174
0209211
+ in progress

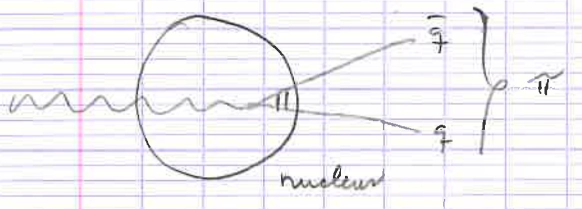
Within every nonabelian gauge theory is hidden QG.

$\uparrow \ominus \uparrow \overset{?}{=} \Uparrow$

Weinberg Witten thm: (1980): (no go)
local observables: gauge theory yes $\langle \text{graviton} | T_{\mu\nu} | \text{graviton} \rangle$
gravity no

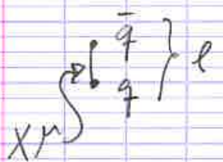
assumption
$g_{\mu\nu}$ lives in same spacetime as A_μ

color transparency:



$\pi \Leftrightarrow$ small dipole

dipoles interact only weakly with long wavelength fields



" in 5th dimension π ion leaves "

1) More dof $N_c \to$ large $g^2 N_c =$ fixed
2) $g_+^2 N_c$ large (
3) SUSY so coupling can get strong $H = Q^2 \geq 0$ $N=4$

$\exists \Lambda \ell$

$\ell \to \lambda \ell$ ~~$x \to \lambda x$~~ $x \to \lambda x$

invariant metric: $\left(\frac{R'}{\ell}\right)^2 dx^\mu dx^\nu \eta_{\mu\nu} + \frac{R^2}{\ell^2} d\ell^2 \Big) = R^2 dz^2 + \frac{\eta_{\mu\nu} dx^\mu dx^\nu}{z^2}$

$z = \frac{R}{R'} \ell$

II B supergravity

$R_1 = m_s N_c^{1/4}$

$\mathcal{N} = 4 \quad A_\mu \oplus 6 \times \chi \oplus 4 \times \psi$

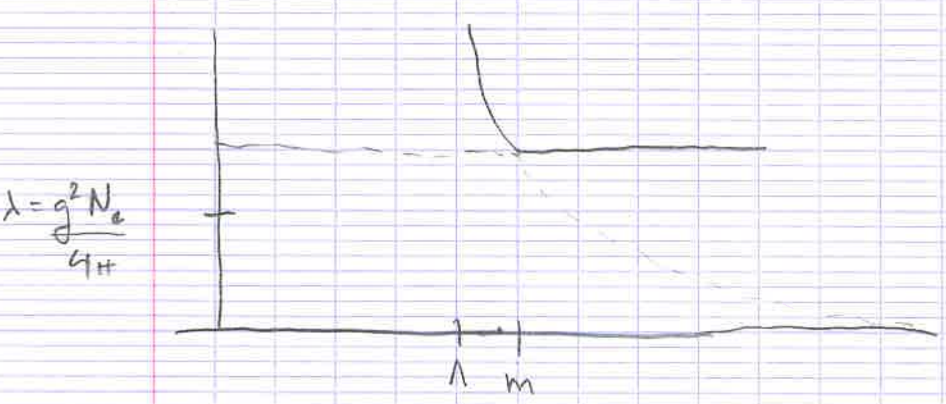
$$\overset{\Delta=3}{} \qquad \overset{\Delta=2}{}$$

$$H = H_{\mathcal{N}=4}^{*} + m_\alpha \bar{\psi}_\alpha \psi_\alpha + m_\alpha^2 \chi_\alpha^2$$

(local gauge invariant operators correspond to bulk modification by dipole argument)

$E \gg m \quad \mathcal{N} = 4$

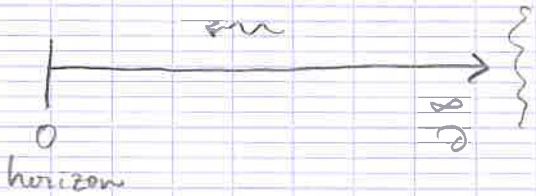
$E \ll m \quad \mathcal{N} = 0, 1, 2$



$r = \frac{R^2}{z}$

GKP, W 1998

$$ds^2 = \left(\frac{r}{R}\right)^2 \eta_{\mu\nu} dx^\mu dx^\nu + \frac{R^2}{r^2} dr^2$$



$H = H_{\mathcal{N}=4} + g\,\mathcal{O}$

$\mathcal{O}$ = dimension Δ

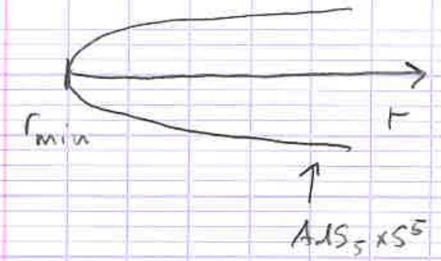
δ fields $\sim r^{\Delta - 4}$

$\Delta = 4$ marginal

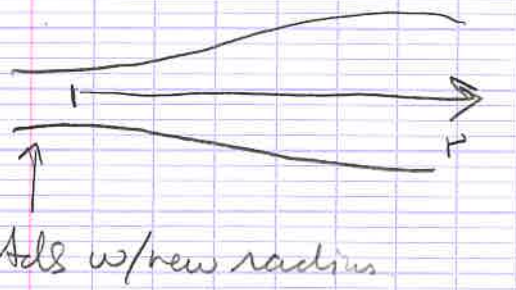
$\Delta > 4$ non renormalizable (?) Gubser, Hashimoto

$\Delta < 4$ relevant

F_{MNP}, H_{MNPQ} all indices along the S^5



$$\frac{r^2}{R^2}\eta_{\mu\nu}dx^\mu dx^\nu$$



radial domain wall.

Wilson loops added?

$r_{min} \rightarrow$ mass gap $\rightarrow$ confinement

$$E_{4D} = \frac{r}{R} E_{10D} \sim \frac{r}{R^2}$$

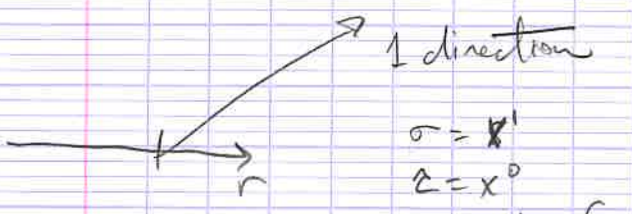
$$\downarrow$$

$$i\partial_{t_a}$$

$$\sqrt{\tau_{H}} = \frac{r}{R^2}\lambda^{1/4}$$

$r > r_{min}$

$$E > \Lambda = \frac{r_{min}}{R^2}$$



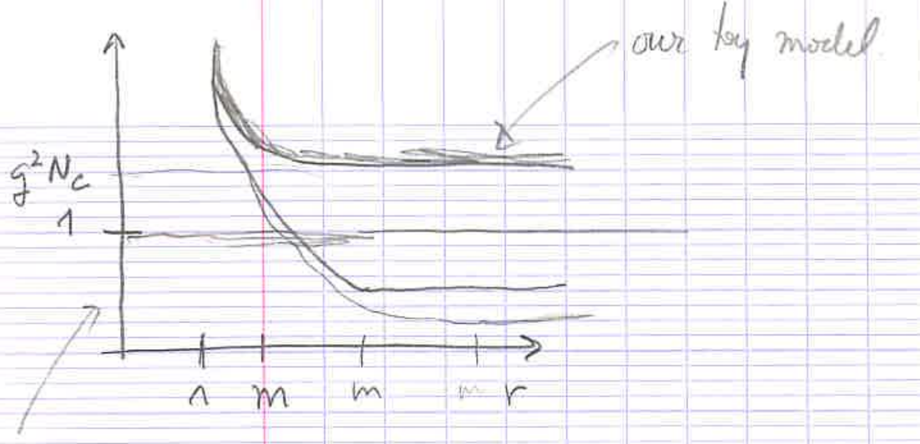
$\sigma = x^1$

$\tau = x^0$

$$S_{WS} = \frac{1}{4\pi\alpha'}\int d^2\sigma\, G_{\mu\nu}\,\partial_a X^\mu \partial^a X^\nu$$

$$\swarrow$$

$$\frac{r^2}{4\pi\alpha' R^2}\int d^2\sigma\, \eta\, \partial X \partial X$$



below 1
string is
strongly coupled.

Klebanov Strassler $d=4$
Maldacena Nunez $d=6$
Witten (thermal circle) $d=5$

X scattering

X transparent prop of gauge plasma = of black horizon